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The translation of isolated cold eddies on a sloping bottom

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Abstract—A two-layer analytical model is considered to examine the dynamics of cold isolated patches on the ocean floor. Such patches have been observed in the North Atlantic Ocean and are characterized by a bounding interface that intersects the bottom along a closed curve. They correspond, therefore, to isolated anticyclonic eddies with a lens-like cross section. The model incorporates steady movements resulting from the swirl velocity within the eddy and a topographically-induced translation.

The movements are assumed to be frictionless and nondiffusive but are not restricted to be quasi-geostrophic in the sense that the Rossby number is not necessarily small. For steady motions, analytical solutions are obtained using the full equations of motion in a coordinate system moving with the eddy itself.

A uniformly sloping bottom causes a steady translation at 90° to the right of the downhill direction. Thus, the model predicts that an anticyclonic eddy on the ocean floor will migrate along lines of constant depth. Surprisingly, the predicted translation speed depends only on the gravitational acceleration, the density difference between the layers, the Coriolis parameter, and the bottom slope. It is independent of the intensity, size, and depth of the eddy.

For the range of parameters typical for the deep ocean, the predicted translation speed is 5 to 10 cm s^{-1}. It is estimated that isolated eddies on the ocean floor may be able to carry temperature anomalies for a few thousand kilometers away from their origin.

1. INTRODUCTION

ISOLATED baroclinic eddies are found in many parts of the ocean. In the upper ocean and at mid-depth they result from instabilities of currents and from various outflows such as the Mediterranean (MCDOWELL and ROSSBY, 1978; NOF, 1982) and the Amazon (RYTHER, MENZEL and CORWIN, 1967; NOF, 1981a). So far, there have been few observations of isolated eddies on the deep-ocean floor, but recently ARMI and D'ASARO (1980) identified isolated cold patches on the abyssal plane, and in April 1977 (during POLYMODE), a cold isolated 'blob' was observed on the bottom near 70°W and 25°N (EBBESMEYER, personal communication).

In a similar fashion to upper-ocean anticyclonic eddies, the blobs are characterized by a lens-like cross section corresponding to an interface that intersects the bottom along a closed curve. They have various length and temperature scales; the patches identified by ARMI and D'ASARO (1980) had a temperature anomaly of ~0.05°C, a depth of ~40 m, and a length of ~20 km. The feature identified during POLYMODE was considerably larger; it had a depth of ~300 m, a length of ~200 km, and a temperature anomaly of ~0.4°C.

Such blobs probably play an important part in determining the mixing and distribution of heat in the deep ocean, so there is an interest in examining their dynamical behavior. The

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question of their movement and migration on the ocean floor is of particular importance because of its relevance to the transport of properties and energy within the deep ocean.

In the upper ocean, mesoscale eddies translate due to the variation of the Coriolis parameter with latitude (see e.g., FLIERL, 1977, 1979b; FLIERL *et al.*, 1980; MIED and LINDEMANN, 1979; MCWILLIAMS and FLIERL, 1979; WARREN, 1967; NOF, 1981b). In view of this and the fact that a sloping bottom is usually thought of as being equivalent to the variation of the Coriolis parameter with latitude in the upper ocean, it is expected that isolated cold eddies will translate in most of the deep ocean. To investigate the possibility of such movements, we shall consider a two-layer model of an isolated anticyclonic eddy on a sloping bottom.

Our aim is to examine the general behavior of isolated cold eddies on an inclined floor; no attempt will be made to produce detailed models for particular sets of observations such as those of EBBESMEYER (personal communication) or ARMI and D'ASARO (1980). We shall focus our attention on the possible translatory movements and show that, as expected, a sloping bottom causes a translation of the whole eddy similarly to the effect of β in the upper ocean. We shall see, however, that although both processes are associated with translatory movements, the magnitude of the topographically-induced movement and its various properties are entirely different from those associated with β-induced movement in the upper ocean. It will be shown that the topographically-induced translation is relatively fast and independent of the intensity, size, depth, and volume of the eddy.

The governing equations are considered in a coordinate system moving with the eddy itself, because it turns out that with such a coordinate system the translation speed can be readily calculated by integrating the governing equations over the whole eddy. The integrated equations relate the translation of the eddy to the slope of the bottom and to the density difference between the layers.

This paper contains the formulation of the problem (Section 2) and its solution (Section 3). Section 4 contains the analysis of the results and their limitations, and Section 5 is a summary.

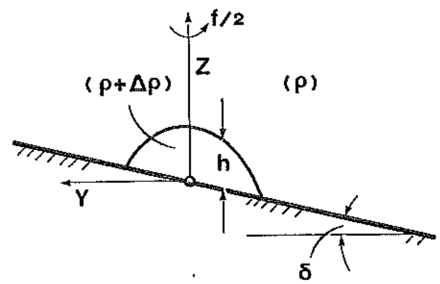
2. FORMULATION

As an idealized formulation of the problem, consider the structure shown in Fig. 1. The cold, isolated eddy has an homogeneous density $(\rho + \Delta\rho)$ and depth $h(x, y)$. The infinitely deep fluid above the eddy has a density (ρ) and is taken to be at rest.

Let us suppose now that the eddy has been suddenly placed on the sloping bottom. Shortly afterwards, the gravitational force will tend to cause sliding of the whole eddy in the downhill direction. Such movement, however, will be modified by the Coriolis force, which will tend to deflect the whole eddy toward the right until a balance of forces is reached. It is this final balanced state we shall examine. To do so, we shall consider a coordinate system moving with the eddy, which eliminates the time dependency of a steadily translating eddy.

Before considering the equations of motion for such a moving coordinate system, it is useful to examine some general properties of cold anticyclonic eddies using a stationary frame of references. In a fixed coordinates system, the governing equations for a frictionless Boussinesq fluid are

$$\frac{\partial \mathbf{u}_s}{\partial t} + \mathbf{u}_s \cdot \nabla_H \mathbf{u}_s + f\mathbf{k}x\mathbf{u}_s + \frac{1}{\rho}\nabla_H p = 0 \tag{2.1}$$



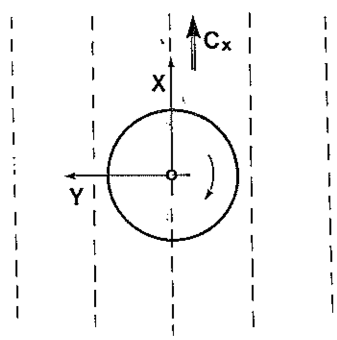


Fig. 1. Schematic diagram of the model under study. The isolated anticyclonic eddy is placed on a sloping bottom ($S = \tan\delta$; $S \ll 1$); the upper layer, whose density ρ is slightly lighter than that of the eddy ($\rho + \Delta\rho$), is infinitely deep. Dashed lines denote lines of equal depth. The eddy is translating steadily at a speed C_x along the lines of constant depth.

$$\frac{\partial h}{\partial t} + \nabla_H \cdot (h\mathbf{u}_s) = 0. \tag{2.2}$$

Here, the subscript s denotes that the variable in question is considered in a stationary frame of references, $\mathbf{u}_s$ is the horizontal velocity vector, ρ the density, p the deviation of the pressure from its motionless hydrostatic value, $\mathbf{k}$ a unit vector in the z direction, f the Coriolis parameter (taken to be constant), ∇_H the horizontal del-operator, and h is the eddy depth. The $\hat{x}$ and $\hat{y}$ axes are directed at 90 and 180° to the left of the downhill direction, and the origin is (x_0, y_0). It has been assumed that all motions are hydrostatic so that the horizontal velocity components are independent of depth and (2.1) and (2.2) are equivalent to the so-called shallow-water equations.

If there is no slope to the bottom the eddy is stationary and $\partial/\partial t \equiv 0$. By writing (2.1) and (2.2) (with $\partial/\partial t \equiv 0$) in polar coordinates, examining the structure of the corresponding potential vorticity equation, and considering the condition that at the outer streamline of the eddy the depth vanishes, one can easily show that the system always has radially symmetric solutions corresponding to purely circular motion. This means that on an f plane and a flat bottom an isolated anticyclonic eddy free from any external forcing is always circular. It will become clear later that, in our problem, the presence of translation does not alter the shape of the eddy, so the translating eddies will also be circular.

With the aid of this general information about the shape of the eddy we shall examine the problem in the moving coordinate system. The origin of the system is at the center of the eddy, the x axis is directed at 90° to the left of the downhill direction, the y axis is pointed uphill, and the system rotates uniformly with an angular velocity $\frac{1}{2}f$ about the vertical axis (Fig. 1). It is assumed that the translation is steady and that the shape of the eddy is permanent and does not change in time so that in our moving coordinate system the motion appears to be steady. The assumption of permanent form and structure is plausible, but it is not *a priori* obvious under what conditions it is valid and adequate. It will be demonstrated, however, that the translation does not affect the structure of the eddy in any way, so the shape is permanent, and the assumption is adequate for all topographically-induced translations.

The relevant governing equations for the moving coordinate system are obtained by applying the transformations $\hat{x} \to x + C_x t$ and $\hat{y} \to y$ to (2.1) and (2.2). For the conditions mentioned above, the transformed equations are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{2.3}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + f(u + C_x) = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{2.4}$$

$$\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0, \tag{2.5}$$

where u and v are the horizontal velocity components in the x and y direction. Note that the term fC_x on the left-hand side of (2.4) results from the fact that in a moving coordinate system there is an apparent force acting on all fluid parcels. As mentioned earlier, we shall focus our attention on eddies embedded in an infinitely deep fluid. Under such conditions, translatory movement of the eddy has no effect on the upper layer, resulting from the fact that in a moving coordinates system the upper layer behaves as if it were flowing over a bump whose height is $h(x, y)$. Thus the disturbances are proportional to (h/H), where $H \to \infty$ (see e.g., INGERSOLL, 1969; NOF, 1982). Hence, the disturbances vanish and the eddy travels without affecting the fluid above. The situation would have been quite different had the upper layer not been infinitely deep. With a finite upper layer there might be important interactions between the eddy and its surroundings so that the subsequent analysis may not be applicable (Section 4).

Under the conditions mentioned above, the deviation of the hydrostatic pressure from the hydrostatic pressure associated with a state of rest (i.e., a 'no blob' state) is

$$p = g\Delta\rho\,[h(x,y) + Sy - z], \tag{2.6}$$

where S, the slope of the bottom, must be small so that the hydrostatic assumption is not violated. Substitution of (2.6) into (2.3) and (2.4) gives

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-fv=-g'\frac{\partial h}{\partial x} \tag{2.7}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+f\left(u+C_x+\frac{g'S}{f}\right)=-g'\frac{\partial h}{\partial y}, \tag{2.8}$$

where g' is the 'reduced gravity' defined by $g'=(\Delta\rho/\rho)g$.

The set (2.7) and (2.8) is subject to the following boundary conditions:

$$h=0:\ \phi(x,y)=0 \tag{2.9}$$

$$(u\mathbf{i}+v\mathbf{j})\cdot\nabla_H\phi=0;\quad \phi(x,y)=0, \tag{2.10}$$

where $\phi(x,y)=0$ denotes the outer edge of the eddy. Condition (2.9) states that $h=0$ along a curve not known in advance (ϕ) and (2.10) requires that the edge be a streamline. The conditions correspond to the fact that the location and shape of the outer edge of the eddy are not known *a priori* but must be determined as part of the problem.

3. SOLUTION

To obtain the solution to the problem, (2.8) is integrated over the entire eddy:

$$\iiint_V\left(u\frac{\partial v}{\partial x}+v\frac{\mathrm{d}v}{\mathrm{d}y}\right)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z+\iiint_V fu\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

$$+\iiint_V\left(C_x+\frac{g'S}{f}\right)f\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z=-g'\iiint_V\frac{\partial h}{\partial y}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,$$

where V denotes the volume of the vortex. As u and v are independent of z, the integrated balance reduces to

$$\iint_A\left(hu\frac{\partial v}{\partial x}+hv\frac{\partial v}{\partial y}\right)\mathrm{d}x\,\mathrm{d}y+\iint_A fuh\,\mathrm{d}x\,\mathrm{d}y$$

$$+\iint_A\left(C_x+\frac{g'S}{f}\right)fh\,\mathrm{d}x\,\mathrm{d}y=\frac{-g'}{2}\iint_A\frac{\partial}{\partial y}(h^2)\,\mathrm{d}x\,\mathrm{d}y, \tag{3.1}$$

where A denotes the projection of the outer surface of the eddy on the x,y plane. Using (2.5) and Stokes' theorem, (3.1) can be expressed as

$$-\int_\phi hv^2\,\mathrm{d}x+\int_\phi huv\,\mathrm{d}y+\iint_A fuh\,\mathrm{d}x\,\mathrm{d}y+\iint_A\left(C_x+\frac{g'S}{f}\right)fh\,\mathrm{d}x\,\mathrm{d}y=\frac{g'}{2}\int_\phi h^2\,\mathrm{d}x, \tag{3.2}$$

where ϕ denotes the outer edge of the eddy (2.9). Because $h=0$ along ϕ, the line integrals over the nonlinear terms and the right-hand side of (3.2) vanish identically. With the aid of the transport function

$$\frac{\partial \Psi}{\partial y} = -uh; \quad \frac{\partial \Psi}{\partial x} = vh, \tag{3.3}$$

the resulting equation can be written in the form

$$-\iint_A \frac{\partial \Psi}{\partial y} \, dx \, dy + \iint_A \left(C_x + \frac{g'S}{f} \right) h \, dx \, dy = 0, \tag{3.4}$$

which, by using Stokes' theorem again gives

$$\int_\phi \Psi \, dx + \iint_A \left(C_x + \frac{g'S}{f} \right) h \, dx \, dy = 0. \tag{3.5}$$

As Ψ = constant along ϕ (see 2.10), the first integral on the left-hand side also vanishes. Therefore, (3.5) reduces to the simple relationship

$$C_x = -g' \frac{S}{f}, \tag{3.6}$$

which states that the translation speed depends only on the reduced gravity, the slope of the bottom, and the Coriolis parameter. When the procedure is applied to equation (2.7), one finds that all the integrals vanish identically indicating that the integrated forces in the x direction are in balance for any translation speed. In the deep ocean, the typical numerical values for g', S, and f are $g' \sim 2 \times 10^{-4}$ m s^{-2}, $S \sim 0.02$, and $f \sim 10^{-4}$ s^{-1}, so the predicted translation speed is $\sim$4 cm s^{-1}, which is comparable to other flows near the ocean floor. The predicted speed (3.6) differs markedly from the β-induced translation in the upper ocean because the latter strongly depends on the properties of the eddy (e.g., NOF, 1981b, equation 3.6) whereas the former does not.

With the aid of (3.6) we can find the general behavior of the interior of the eddy. To do so, (3.6) is substituted back into (2.7) and (2.8) and the resulting equations are written in polar coordinates

$$v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} - f v_\theta = -g' \frac{\partial h}{\partial r} \tag{3.7}$$

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} + f v_r = -g' \frac{\partial h}{\partial \theta} \Big/ r, \tag{3.8}$$

where v_θ and v_r are the tangential and radial velocities. The continuity equation takes the form

$$\frac{\partial}{\partial r} (r v_r h) + \frac{\partial}{\partial \theta} (v_\theta h) = 0. \tag{3.9}$$

The system of equations is independent of C_x showing that the interior structure is not affected by the translation. It can be shown that with the boundary conditions (2.9) and (2.10) the system always has radially symmetric solutions (i.e., $v_r = 0$; $\partial/\partial\theta = 0$). Such solutions are similar to the known solutions for upper-ocean eddies with uniform potential vorticity (CSANADY, 1979; FLIERL, 1979a). Radially symmetric solutions for varying potential vorticity can also be derived for (3.7) to (3.9), and an example of such a case is considered in Section 4. Whether the potential vorticity is uniform or not, solution (3.6) and the system (3.7) to (3.9) show that the translation speed does not depend on the eddy interior, the interior structure is not affected by the translation, and the interior solutions correspond to circular eddies with purely tangential movements.

The independence of the topographically-induced translation (C_x) on the intensity, depth, and volume of the eddy results from the fact that the eddy migrates as if it were a solid body with a density ($\rho + \Delta\rho$). This can be demonstrated by considering the gravitational force (F_g), which acts in the downhill direction

$$F_g = -g\Delta\rho S \iint_A h \,\mathrm{d}x\,\mathrm{d}y \tag{3.10}$$

and the integrated Coriolis force (F_c), which acts on the whole eddy

$$F_c = \rho f C_x \iint_A h \,\mathrm{d}x\,\mathrm{d}y. \tag{3.11}$$

Both forces are independent of the swirl speed and are linearly proportional to the volume of the eddy. If the eddy translates as if it were a solid body, the forces should be in balance (i.e., $F_c = F_g$). Such a balance requires $C_x = -g'S/f$, which is identical to the result (3.6) obtained by integrating the full equations for fluid motion (2.7) to (2.8) indicating that the fluid circulating within the eddy has no effect on the topographically-induced movement.

The fact that the predicted translation speed (3.6) is independent of the intensity, size, or volume of the eddy is of fundamental importance. It means that a group of isolated eddies of different sizes and strengths but equal densities will travel at a uniform speed (Fig. 2). Similarly, a pack of isolated eddies which are very close to each other (but not touching) will also translate at $-g'S/f$.

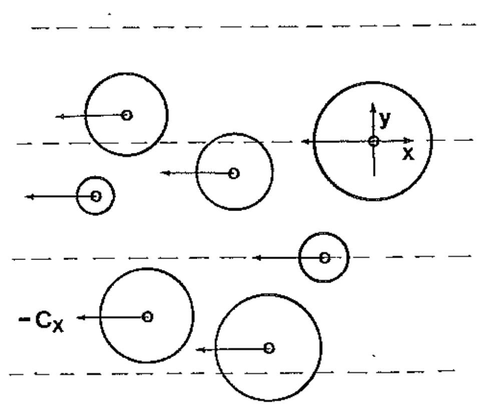


Fig. 2. A sketch of a group of isolated eddies, with different sizes and intensities but with equal densities, translating on a sloping bottom. All eddies translate with the same speed ($C_x = -g'S/f$) at 90° to the right of the downhill slope. Dashed lines indicate lines of equal depth; the depth decreases with y. The uniform translation speed results from the fact that the translation is independent of the properties of the eddy.

4. ANALYSIS

Validity of assumptions

The foregoing theory relied on the assumption that the upper layer is infinitely deep. However, our result is adequate not only for an infinitely deep upper layer but also for some upper layers with a finite depth. To illustrate this point, we shall consider the potential vorticity for an upper layer with a finite depth ($H - Sy$; $S \ll 1$) in the same coordinate system used earlier (i.e., moving with the eddy at its own speed). In this system, the upstream upper layer appears to the observer to be flowing toward the eddy at a speed C_x, and the problem resembles a uniform homogeneous flow approaching an isolated bump. The corresponding potential vorticity equation is

$$\frac{D}{Dt}\left(\frac{\partial v'/\partial x - \partial u'/\partial y + f}{H - Sy - h + \eta}\right) = 0, \quad \frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \tilde{u} \cdot \nabla \tilde{u}, \tag{4.1}$$

where $\tilde{u} = u'\mathbf{i} + v'\mathbf{j}$. Here, $H - Sy$ is the undisturbed depth of the upper layer, u' and v' are the horizontal velocity components induced by the eddy, which acts as obstacle, h is the eddy depth, and η is the free surface vertical displacement ($\ll h$). Upstream, $u' = v' = 0$ so that the potential vorticity is $\sim f/H$. Therefore,

$$v' \sim u' \sim 0(fhl/H), \tag{4.2}$$

where l is the length of the eddy. If $H \to \infty$, then $u' \to 0$ and $v' \to 0$, showing that for an infinitely deep fluid the condition of a freely translating eddy is certainly adequate. If the upper layer depth is finite, then u' and v' are not identically zero and, for our solution to be valid, we must require that $|u'| \ll |C_x|$ and $|v'| \ll |C_x|$. Hence, condition (4.2) gives

$$1 \ll \frac{g'HS}{f^2 lh} \tag{4.3}$$

as being the condition for negligible influence of the flow in the upper layer on the translation speed.

Condition (4.3) is satisfied by many, but not all, isolated cold eddies. For instance, if $\Delta\rho/\rho \sim 2 \times 10^{-5}$, $H \sim 5{,}000$ m, $S \sim 0.04$, $f \sim 10^{-4}$ s^{-1}, $l \sim 20$ km, and $h \sim 20$ m, then the right-hand side of (4.3) is about 10 so the perturbations in the upper layer have a small effect on the translation speed as required. However, if $h \sim 200$ m and all other scales are similar to those above, the right-hand side of (4.3) is of order unity and the condition is not met.

Stability

The physical reality of our analysis depends on the stability of the eddies. A detailed examination of the stability is beyond the scope of this study but some aspects of the problem can be examined by considering the magnitude of the Richardson number ($Ri \equiv g'h/v_\theta^2$) near the rim. It is clear that when the orbital velocity is non-zero along the 'front' (i.e., where the interface intersects the floor, $h = 0$) then the Richardson number is smaller than 1/4 in a ring confined between the edge of the eddy and a smaller circle with a radius of, say, $\tilde{r}$. In this region local instability will probably occur; it can be avoided only if $v_\theta \to 0$ as $h \to 0$.

Aside from such local instability, the whole eddy may be baroclinically unstable (under certain conditions) as suggested by GRIFFITHS and LINDEN (1981) and GRIFFITHS, KILLWORTH and STERN (1982). However, even if the eddy is baroclinically unstable and, consequently, breaks into smaller eddies, the translation speed will not be affected because it is independent of the properties of the eddy. In other words, the resulting smaller eddies will continue moving at the same speed as the parent eddy because the size and depth have no influence on the migration.

Interior structure and absolute velocities

As the migration is associated with orbital movements it is of interest to examine the total movement that a fluid parcel will have. For this purpose we shall consider the simplest possible eddy that obeys (3.7) to (3.9). We shall assume that the swirl velocity (v_θ) is parabolically distributed, i.e.,

$$v_\theta = 2Rofr\left(\frac{r}{r_0} - 1\right), \tag{4.4}$$

where r_0 is the radius of the eddy and Ro is the Rossby number defined on the basis of the maximum swirl velocity and its distance from the center ($r_0/2$). There is an upper limit on the magnitude of the Rossby number that can be associated with (4.4). It cannot assume values larger than 1/4 because otherwise the negative relative vorticity would be larger than f (as $r \to 0$), which is impossible.

For the present discussion it is not important to know how (or why) such an eddy has been formed because we are merely interested in its behavior on a sloping bottom. The depth distribution, corresponding to (4.4), is found with the aid of (3.7) (with $v_r = 0$ and $\partial/\partial\theta = 0$) to be

$$h = \hat{h} + Rof^2r^2(2Ro - 1)/g' + 2Rof^2r^3(1 - 4Ro)/3g'r_0 + Ro^2f^2r^4/g'r_0^2, \tag{4.5}$$

where $\hat{h}$ is the maximum depth of the eddy (i.e., at $r = 0$). The potential vorticity $P(r)$ is

$$P(r) = \frac{1}{h}\left[4Rof\left(\frac{3r}{2r_0} - 1\right) + f\right]$$

which, in contrast to that of the upper-ocean eddies considered by CSANADY (1979) and FLIERL (1979a), is non-uniform and varies with r. In view of (4.5), the eddy radius ($r = r_0$, $h = 0$) is

$$r_0 = \left[\frac{3g'\hat{h}}{Ro(1 - Ro)}\right]^{1/2} \Big/ f. \tag{4.6}$$

Hence, the ratio between the deformation radius (R_d) and the radius of the eddy is given by

$$\frac{R_d}{r_0} = [Ro(1 - Ro)/3]^{1/2}.$$

The relationship is shown in Fig. 3, which illustrates that the eddy radius is usually ($Ro \ll 1$), much larger than the deformation radius. Even if the Rossby number assumes its highest possible value ($\frac{1}{4}$), the eddy radius will be four times larger than the deformation radius.

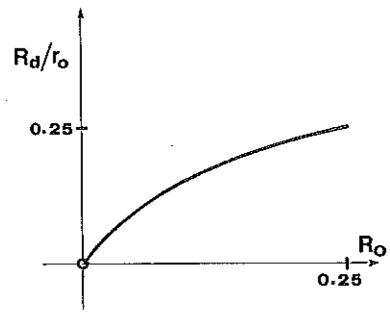


Fig. 3. The dependence of the ratio between the deformation radius (R_d) and the eddy radius (r_0) on the Rossby number (Ro).

With the aid of (3.6) and (4.4) the total velocity components can be written as

$$\left.\begin{aligned} u_a &= -\frac{g'S}{f} - 2Rofr\left(\frac{r}{r_0} - 1\right)\sin\theta \\ v_a &= 2Rof_0 r\left(\frac{r}{r_0} - 1\right)\cos\theta, \end{aligned}\right\} \tag{4.7}$$

where the subscript a indicates that the variable in question corresponds to the absolute value relative to a fixed point in space. As there is no limitation on the ratio between the translation speed and the orbital velocity, the migration can be faster than the swirl speed so that u_a can be negative for all values of r. Simple scaling suggests that such would be the case whenever $S > \hat{h}/r_0$. For instance, if $\hat{h} \sim 20$ m, $r_0 \sim 20$ km, $g' \sim 2 \times 10^{-4}$ m s^{-2}, $S \sim 0.04$, and $f \sim 10^{-4}$ s^{-1}, then $C_x \sim -0.08$ m s^{-1} whereas the u component resulting from the orbital movement $[2Rofr(r/r_0 - 1)\sin\theta]$ is smaller, about 0.002 m s^{-1}.

Possible frictional effects

Because the eddy is translating on a solid bottom, frictional effects are expected to be larger than those of upper-ocean eddies and interior eddies. For this reason, it is important to examine, at least qualitatively, some of the effects expected in real fluids.

One intuitively expects to find two major effects of dissipative mechanisms. The first is that of friction on the swirl speed, which will slow down and thus will tend to flatten out the eddy. The second is the effect of friction on the translatory movement of the entire eddy.

The spin-down effect operates on large time scales; for interior and upper-ocean eddies its time scale is years (see e.g., GILL, 1980) and for the eddies under consideration it is probably months. The effect of a continuous reduction in the swirl speed on the translation speed is probably fairly limited because, as we saw earlier, the calculated translation speed (3.6) is independent of the swirl speed.

In contrast to the above limited effect, the influence of the component of the bottom stress that opposes the translation will probably be important. The presence of such an opposing force will cause a change in the direction of translation because there is no force that can balance it if the eddy continues to move steadily in the x direction (i.e., along lines of constant depth). It can only be balanced if the eddy changes its direction from a pure movement in the

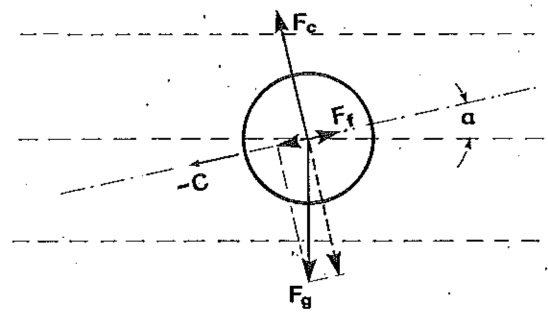


Fig. 4. A schematic diagram of the possible balance of forces when frictional forces are present. The gravitation force (F_g) has two components; one balances the Coriolis force acting on the whole (F_c), and the second balances the opposing frictional force (F_f). Dashed lines denote lines of equal depth. As a result of friction, the translation takes place along lines of increasing depth instead of lines of equal depth. The angle α represents the direction of translation.

negative x direction to a direction including a downhill component (Fig. 4). Under such conditions the downhill force resulting from the weight of the eddy has two components. One balances the Coriolis force, which acts on the eddy as a whole, and the other balances the frictional force, which acts in the direction opposing the translation (F_f). That is,

$$fC\iint_A h \, dx \, dy = g'S \cos \alpha \iint_A h \, dx \, dy$$

and

$$F_f = g'S \sin \alpha \iint_A h \, dx \, dy,$$

where α is the angle at which the eddy is translating, $C = (C_x^2 + C_y^2)^{1/2}$, and F_f is the unknown frictional force.

It is, therefore, expected that due to bottom friction, the eddy will not move along lines of constant depth but rather along lines of slightly increasing depth (Fig. 4). A loss of potential energy is associated with such motion; the loss corresponds to the work done against the bottom stress. However, while such considerations of possible frictional effects are plausible, they are speculative and a detailed analysis is required to find the complete response to friction.

5. SUMMARY AND CONCLUSIONS

It is appropriate to repeat the limitations involved in our analysis. The most important assumptions that have been made are that the eddy is homogeneous (in the sense that its density does not vary), that the slope of the bottom is uniform over the length of the eddy, that frictional effects are negligible, and that the depth of the eddy is small so that its translatory movement does not cause large disturbances in the fluid above. It has been demonstrated that the assumptions are valid in various cases of practical interest, but in many other cases they may be violated.

The results of the study can be summarized: (1) A sloping bottom causes a translation of isolated cold eddies in a direction 90° to the right of the downhill slope. (2) As the isolated eddy is translating, its whole mass anomaly is carried along and none is left behind. (3) The predicted translation speed is given by $g(\Delta\rho/\rho)S/f$, where $\Delta\rho$ is the density difference between the layers, S is the bottom slope, and f is the Coriolis parameter. (4) Conclusion (3) is valid for all eddies, independently of their volume, intensity, nonlinearity, size, and height. (5) It is estimated that, due to a sloping bottom, deep-ocean eddies may migrate as fast as 5 to 10 cm s^{-1} and may travel as much as a few thousand kilometers away from their origin.

The conclusions demonstrate that isolated cold eddies on a sloping bottom have a unique behavior differing substantially from other eddies in the ocean. Much work, both observational and theoretical, is suggested by our present investigation; the effects of continuous stratification, steering currents, and eddy–eddy interactions are a few examples.

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