Geostrophic Pumping, Inflows and Upwelling in Barrier Reefs

DORON NOF* AND JASON H. MIDDLETON

School of Mathematics, University of New South Wales, Kensington, New South Wales, Australia

(Manuscript received 5 May 1988, in final form 27 December 1988)

ABSTRACT

The communication between shallow and deep oceans via gaps in the separating barrier reefs is examined using a simplified two-layer analytical model. Attention is focused on the flow resulting from a sea-level difference between the ocean and the lagoon. Such a difference imposes a pressure gradient along the gap which, in turn, forces a flow into the lagoon. The coral reefs, which extend all the way to the surface and are exposed to the atmosphere at low tide, are represented by two portions of an infinitely long wall. A group of passages, whose combined width is not very small compared to the Rossby radius, is represented by a single gap separating the two portions of the wall.

The fully nonlinear model is inviscid, hydrostatic and nondiffusive. Nonlinearity is essential because (i) the flow in the passages is rather fast, and (ii) the depth variations are of order unity. Steady solutions for the upstream and downstream fields are constructed analytically using the momentum equation in an integrated form, the Bernoulli integral and conservation of potential vorticity.

It is found that, surprisingly, the transport through the gap is independent of the gap's width. Upstream, the oceanic water approaches the gap only from one direction; upon reaching the gap, the approaching current splits into two branches. One continues to flow in the oceanic basin and never enters the gap whereas the other passes through the gap and penetrates into the lagoon. When the sea-level difference between the ocean and the lagoon exceeds a critical value, water below the oceanic thermocline is pulled up and forced into the lagoon. This nutrient-rich upwelled water forms a boundary current that hugs the barrier reef on the right hand side in the Northern Hemisphere. We term this new type of upwelling and suction "geostrophic pumping" because it is a result of the geostrophic flow away from the gap.

A possible application of this geostrophic pumping to the upwelling and inflow in the Great Barrier Reef is briefly discussed. The model provides a plausible explanation for the health of the coral on the lagoon side where, without such an inflow, the nutrients would have been depleted.

1. Introduction

When dense coral grows on the shelf break, it sometimes forms an almost solid barrier between the ocean and the shelf. Such a situation exists in the coral reefs off Belize and in the Great Barrier Reef off Australia where the coral grows all the way to the surface and even protrudes to the atmosphere at low tide (Fig. 1). The coral forms long chains [O (100 km)] separated by groups of gaps with a combined width of as much as 10–15 km.

The coral on the lagoon side is healthy, suggesting that upwelling must somehow replace the nutrient-poor lagoon water. In this paper a new inflow mechanism that might be relevant to the above upwelling conditions is proposed.

a. Background

The general upwelling question on the shelf off east Australia was addressed by Garrett (1979) who suggested that, in a similar fashion to the upwelling considered by Hsueh and O'Brien (1971), Ekman layers on the bottom drive deep water to shallow depth. Wolanski and Pickard (1983) suggest that the interaction of barotropic tidal currents and bottom topography may be important to the introduction of nutrients to the outer reef. Griffin and Middleton (1986) discuss the role of coastal trapped waves in the upwelling mechanism.

While these studies are informative, they do not address the specific question of upwelling and inflow into lagoons that are partially blocked from the outer shelf. This particular issue was first addressed by Thompson and Golding (1981) and then by Thomson and Wolanski (1984), both of whom considered a fluid withdrawal mechanism related to the so-called Bernoulli-suction (e.g., see Whitehead 1980). It relies on the presence of high tidal speeds which are caused by the passages' funnel-like structure. A similar mechanism
was considered by Stommel et al. (1973) for the Mediterranean Sea.

Both Thompson and Golding (1981) and Thomson and Wolanski (1984) neglect the effect of the Coriolis acceleration, and assume that strong tidal currents are the only driving mechanism. We suggest here a new upwelling mechanism that is not a result of strong tidal inflow but rather a consequence of small long-term sea-level fluctuations. Sea-level oscillations of period 15–30 days and amplitude 10–20 cm have been observed by one of us (JHM) in the Coral Sea directly adjacent to the Great Barrier Reef at Raine Island (12°S and Hydrographers Passage (20°S). These fluctuations, which are associated with the neap tide regime (corresponding to weak tidal flow in the passages), would result in the Coral Sea directly adjacent to the Great Barrier Reef being occasionally higher [\(\sim 10 \text{ cm}\)] than the sea level in the lagoon. In other locations similar sea-level differences may occur as a result of various atmospheric conditions. We suggest that such sea-level differences drive cold deep water into the lagoon via a "geostrophic pumping" process. The idea behind this suction is that, when walls and the rotation of the earth are important, uplifting of the thermocline can occur due to geostrophy. Funnelling and geometrical constructions that are required by the so-called Bernoulli suction do not play any role in this new process.

b. Methods

Consider the following situation as an idealized formulation of the problem. Two unbounded basins are
Our aim is to determine the transport through the gap and to examine the changes that will occur upstream in the oceanic basin. It is expected that, after an initial period of adjustment [O($f^{-1}$)], a steady state will be reached because the initially unbalanced system will tend to approach a state of geostrophic balance in a similar fashion to the adjustment process discussed by Rossby (1938). It is also expected that, when the initial sea-level difference between the basins exceeds a certain "critical" value, the interface will rise above the shelf break. Higher sea-level differences will, therefore, draw deep oceanic water into the lagoon. As mentioned, in analogy to the Bernoulli suction, we call this new effect "geostrophic suction" because the suction occurs due to the geostrophy of the upstream and downstream flows.

To obtain the steady solution to the problem, we shall apply the momentum equations in an integrated form, use the Bernoulli integral and consider conservation of potential vorticity. We shall see that this system of equations gives the desired information for both the upstream (oceanic) and downstream (shelf) flow. This approach enables one to construct solutions for the regions away from the gap without finding the detailed solution within the gap itself.

Our approach is similar to the treatment used by Nof and Olson (1983) for the flow through the Windward Passage. A critical difference between the dynamical system of Nof and Olson (1983) and the present analysis is that, in the former the basin into which the light water penetrates is stratified and infinitely deep, whereas in the latter the lagoon is homogeneous and of finite depth. Also, in the present case the system is forced by a free independent sea-level difference, whereas in the former the sea-level difference was an integral part of the problem. As a result of these differences, the present analysis is considerably more complicated. Note that there is some (limited) overlap between the two articles because an attempt has been made to make the present article self-contained. The reader who is familiar with the Nof and Olson article and is not interested in the details may skip sections 2, 3 and 4 and go directly to section 5. After the solutions are presented, the results are qualitatively compared to the upwelling in the Great Barrier Reef. We shall see that the predicted results agree with the location and direction of the observed flows.

This paper is organized as follows. The formulation is discussed in section 2. The solution and its analysis are given in sections 3–7; section 8 contains the application to the Great Barrier Reef and section 9 summarizes this work.

2. Formulation

As an idealized formulation of the problem, consider again the two-layer system shown in Fig. 2. Our model is frictionless, hydrostatic and nondiffusive but the

---

1 Even though the application of our model is to the Great Barrier Reef where $f < 0$, we shall, for convenience, consider positive $f$ for our general analysis. Later on, when the application is specifically discussed, we shall consider $f < 0$. 
motions in the gap and its immediate vicinity are not constrained to be quasi-geostrophic. Full nonlinearity is allowed because the speeds within the gap are expected to be high and the interface displacements are expected to be of order unity.

The origin of our coordinate system is located at the edge of the gap (Fig. 2). The horizontal $x$ and $y$ axes are oriented across and along the gap, and the system rotates (uniformly) about the vertical $z$ axis. Initially, a gate, extending across the gap from the free surface to the shelf break, separates the two-layered oceanic basin from the one-layered lagoon. The initial current in the oceanic basin flows on top of an infinitely deep motionless layer; it has a uniform potential vorticity $f/D_0$ [where $D_0$ is the upper layer undisturbed depth (at $x \to \infty$)]. Consequently, its velocity ($U$) and depth ($D$) are

$$U = U_0 e^{y/R_d}, \quad D = D_0 [1 - (U_0/fR_d)e^{y/R_d}],$$

where $U_0$ is the initial current speed near the wall ($y = 0$) and $R_d$ is the deformation radius $(gD_0)^{1/2}/f$.

Here, $g'$ is the “reduced gravity” $g\Delta \rho / \rho$. The level of the oceanic basin at $y \to -\infty$ is higher than that of the lagoon by $\delta$.

At $t = 0$ the gate is lifted and, subsequently, light fluid starts penetrating into the lagoon. It is expected that, simultaneously, gravity waves will be generated in the gap and its vicinity. These waves will propagate outward away from the gap. Within a period of $O(f^{-1})$, however, the waves in the vicinity of the walls will be modified into Kelvin waves. Since such waves require the presence of a wall on their right, no disturbances can penetrate into regions 2 and 4 (Fig. 3). Consequently, these regions will remain unaltered and will not be influenced by the fact that there is a flow through the gap. It is expected that after some time a steady state will be reached and it is this state that we shall focus on (Fig. 3). Note, however, that as in all adjustment problems (e.g., Rossby 1938), we have no guarantee that the steady state will actually be reached nor are we guaranteed that there is only one steady solution.

Away from the gap, the steady upstream and downstream currents are expected to be in geostrophic balance. Both regions 2 and 4 remain unaltered so that their final fields are identical to the initial fields. Namely, in the final steady state, region 4 will be stagnant and the velocity and depth of region 2 will be identical to the speed and depth that we have started with ($U$ and $D$).

Our flow system consists, therefore, of two unknown boundary currents (regions 3 and 1) and a nonlinear region in the gap and its vicinity. It is not a priori obvious that the resulting downstream and upstream flows (regions 3 and 1) can be connected without solving for the gap and its neighborhood. We shall see, however, that the use of the integrated momentum equation enables one to construct the upstream and downstream fields without looking at the detailed flow structure within the gap. When the sea-level difference $\delta$ reaches a critical value ($\delta_0$), the interface strikes the shelf break (Fig. 4) and any further increase will produce upwelling of deep bottom water (Fig. 5). We will be able to determine the onset of upwelling but, it turns out, that the detailed solution corresponding to the upwelling state shown in the lower panel of Fig. 5 involves a complicated closure condition and it is, therefore, left as a subject for future investigation. We shall briefly return to this point later.

3. General solution for the upstream and downstream regions

We shall first determine the equations governing the flow several deformation radii away from the gap.

a. Region 3

For this region the potential vorticity and geostrophic relation give

---

Fig. 3. Schematic diagrams of the final adjusted state. The flow in region 2 is identical to the initial flow (Fig. 2) because no Kelvin waves can ever reach it. Similarly, region 4 is unaltered and remains stagnant. The free surface displacements ($\eta$) are measured upward from the undisturbed free surface and the interface displacements ($\xi$) are measured downward from the interface undisturbed level. The sea-level difference $\delta$ is measured downward from the oceanic undisturbed free surface to the lagoon undisturbed surface.
because in an inviscid, steady model, there is no mechanism by which momentum can be transferred from the intruding current to the neighboring water.

The general solution for region 3 is

\[ u_3 = f \left( 1 - \frac{H_3}{D_{\infty}} \right) y = u_{3w} \quad \text{(3.4)} \]

\[ -g'h_3 = f u_{3w}(y - \gamma_3) + \frac{f^2}{2} \left( 1 - \frac{H_3}{D_{\infty}} \right)(y^2 - \gamma_3^2), \quad \text{(3.5)} \]

Fig. 4. Schematic diagram of the onset of upwelling. When the sea-level difference \( \delta \) is large enough the interface strikes the shelf break. The sea-level difference corresponding to this transitional state is referred to as \( \delta_u \) (the subscript “u” indicates that the variable in question is associated with upwelling).

\[ -\frac{\partial u_3}{\partial y} + f = \frac{H_3 f}{D_{\infty}} \quad \text{(3.1)} \]

\[ f u_3 = -g' \frac{\partial \eta_3}{\partial y}, \quad \text{(3.2)} \]

where \( u_3 \) is the horizontal velocity component in the x direction, \( H_3 \) is the shelf undisturbed depth, \( \eta_3 \) is the free surface displacement, and the subscript 3 denotes that the variable in question is associated with region 3. Note that the rigid lid approximation has been invoked.

The boundary conditions for region 3 are

\[ \eta_3 = 0; \quad y = \gamma_3 \quad \text{(3.3a)} \]

\[ \left( \frac{u_3^2}{2} \right)_{y=\gamma_3} = \left( \frac{u_1^2}{2} + g'h_1 \right)_{y=0} + g' \delta \quad \text{(3.3b)} \]

\[ \left( \frac{u_3^2}{2} + g'\eta_3 \right)_{y=0} = \left( \frac{u_1^2}{2} + g'h_2 \right)_{y=0} + g' \delta, \quad \text{(3.3c)} \]

where, as before, the subscript 1 indicates association with region 1. Relations (3.3b) and (3.3c) represent the condition that, in the final steady state, energy is conserved along the two streamlines that bound the intruding current from left and right. Both correspond to an application of the Bernoulli principle, which states that the quantity \( (u^2 + v^2)/2 + p/\rho \) (where \( p \) is the pressure), is conserved along the bounding streamlines. The latter (3.3c) is an application of the Bernoulli invariant to the streamline associated with the left wall (looking offshore), whereas the former (3.3b) is associated with its application to the right wall and the free-bounding streamline. Note that beyond the free-bounding streamline \( y = \gamma(x) \) the lagoon is at rest.

Fig. 5. A three-dimensional view of the interface corresponding to the bottom of the oceanic thermocline. The state corresponding to no intrusion into the lagoon is shown in (a). At the onset of upwelling (b) the interface “kisses” the bottom of the lagoon. When the sea-level difference between the ocean and the lagoon increases \( (\delta > \delta_u) \) deep bottom water is upwelled into the lagoon (c).
where condition (3.3a) has been used but (3.3b) and (3.3c) have not been invoked yet. Here, the subscript "w" indicates association with the wall.

b. Region 1

The fluid in this area is also governed by the potential vorticity equation and geostrophy so that

$$-\frac{\partial u}{\partial y} + f = h_1/\rho_\infty$$  \hspace{1cm} (3.6)

$$-fu_1 = g' \partial h_1/\partial y.$$  \hspace{1cm} (3.7)

The boundary conditions for this region are

$$u_1 \to 0, \quad y \to -\infty$$  \hspace{1cm} (3.8a)

$$h_1 \to D_\infty, \quad y \to -\infty,$$  \hspace{1cm} (3.8b)

which state that the velocity decays away from the wall. Equations (3.6) and (3.7) can be combined into

$$\frac{\partial^2 u_1}{\partial y^2} - \frac{u_1}{R_d^2} = 0,$$  \hspace{1cm} (3.9)

where, as before, $R_d^2 = g'D_\infty/f^2$. The latter equation gives

$$u_1 = A_1 e^{y/R_d} + B_1 e^{-y/R_d},$$  \hspace{1cm} (3.10a)

where $A_1$ and $B_1$ are constants to be determined from the boundary conditions. The depth $h_1$ is found from (3.6) to be

$$h_1 = D_\infty \left(1 - \frac{A_1}{fR_d} e^{y/R_d} + \frac{B_1}{fR_d} e^{-y/R_d}\right).$$  \hspace{1cm} (3.10b)

Using (3.8) we immediately find that $B_1 = 0$ so that

$$u_1 = A_1 e^{y/R_d},$$  \hspace{1cm} (3.11a)

$$h_1 = D_\infty \left(1 - \frac{A_1}{fR_d} e^{y/R_d}\right).$$  \hspace{1cm} (3.11b)

The general solutions (3.4), (3.5), and (3.11) contain three unknowns ($u_{hw}, \gamma_3$, and $A_1$). Three equations are, therefore, required to solve the problem. We have not yet used two boundary conditions [(3.3b) and (3.3c)] so that we need an additional equation to close the problem. We shall see in the next section that there are two more constraints—the conservation of momentum flux (flow-force) and mass continuity. One of these relationships (mass continuity) turns out to be redundant, due to the application of the Bernoulli invariant to the bounding streamlines, so that we have in fact the desired number of equations.

4. Constraints

The flow in the lagoon is connected to the flow in the ocean via (3.3b) and (3.3c) but there are two additional equations that the unknown variables must satisfy.

a. Mass continuity

This constraint can be written as

$$\int_0^{\gamma_3} u_3 h_3 dy + \int_0^0 U D dy = \int_\infty^0 u_1 h_1 dy. \hspace{1cm} (4.1)$$

As mentioned previously, it will become clear later that this equation is redundant because, as in Nof and Olson (1983), we have applied the Bernoulli invariant along the two bounding streamlines of the intruding current so that in fact we have already stated that the two edges and walls are streamlines. Although this consideration cannot be easily verified at this stage of the presentation, it will be easy to examine (4.1) once the solution is obtained.

b. Momentum flux

Integration of the x momentum equation over the area bounded by the dashed line in Fig. 6 gives

$$\int \int \int \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) dx dy dz - \int \int \int f v dx dy dz$$

$$+ g \int \int \int \frac{\partial h}{\partial x} dx dy dz = 0, \hspace{1cm} (4.2)$$

where $V$ denotes the entire volume of the region. Since the flow is hydrostatic, the pressure is a linear function of $z$ and the horizontal pressure gradients are a function of $x$ and $y$ alone. Hence, $u$ and $v$ are taken to be independent of $z$ [i.e., $u = u(x, y); \quad v = v(x, y)$] and (4.2) reduces to

$$\int \int \left( hu \frac{\partial u}{\partial x} + vh \frac{\partial u}{\partial y} \right) dx dy - \int \int f v h dx dy$$

$$+ g H_s \int \int \frac{\partial h}{\partial x} dx dy + g' \int \int \frac{\partial h}{\partial x} dx dy = 0, \hspace{1cm} (4.3)$$

![Fig. 6. A diagram of the integration area for the momentum equation. In the lagoon (shaded), the integration area is bounded by the wall, the free streamline and a section across region 3. In the ocean, the integration area extends well beyond the expected decay region (i.e., DE is located several deformation radii away from the walls). It is bounded by sections across region 2 and 1, the walls and the line DE which is parallel to the walls.](image-url)
where $S$ is the projection of $V$ on the $xy$ plane, $S_1$ and $S_2$ are the portions of $S$ situated in the lagoon and ocean, respectively. By using the continuity equation

$$\frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0$$

(4.4)

and the streamfunction $\psi$, defined by

$$\frac{\partial \psi}{\partial y} = -uh; \quad \frac{\partial \psi}{\partial x} = vh,$$  

(4.5)

Eq. (4.3) can be expressed as

$$\int_S \int \left[ \frac{\partial}{\partial x} (hu^2) + \frac{\partial}{\partial y} (hv) \right] dx dy$$

$$- \int_S \int f \frac{\partial \psi}{\partial x} dx dy + gH \int_{S_1} \int \frac{\partial \psi}{\partial x} dx dy$$

$$+ \frac{g^\prime}{2} \int_S \int \frac{\partial}{\partial x} (h^2) dx dy = 0.$$  

(4.6)

Conversion of the surface integrals to line integrals (using Stokes' theorem) gives

$$\oint \phi h udx + \oint \phi hu^2 dy - \oint f \psi dy$$

$$+ gH \oint \phi \eta dy + \frac{g^\prime}{2} \oint \phi h^2 dy = 0,$$  

(4.7)

where $\phi$, $\psi_1$ and $\phi_o$ are the boundaries of $S$, $S_1$ and $S_o$ (Fig. 6) and the arrowed circles indicate that the integrals are taken counterclockwise.

Relation (4.7) can be simplified by noting that (i) $h = H_z$ along OB, (ii) $udy = v dx$ along any streamline, (iii) $u, v \to 0$ as $y \to -\infty$, and (iv) $v = 0$ along the solid walls. For such conditions, (4.7) takes the form

$$\int_D (h_1 u_1^2 + g'h_1^2/2 - f\psi_1) dy$$

$$= \int_E (D^2 u^2 + g'D^2/2 - f\psi) dy$$

$$+ \int_A (h_3 u_3^2 + gh \eta_3 - f\psi_3) dy,$$  

(4.8)

where we have used our definition of $\psi$ (i.e., $\psi = 0$ along OB).

We shall now show that (4.8) can be further simplified by taking into account the condition that the flow is geostrophic in regions 1, 2, and 3. To show this we note that for region 3 the geostrophic relationship can be multiplied by $H_z$ and integrated in $y$ to give

$$f\psi_3 = gh_3 \eta_3 + c_3,$$  

(4.9)

where $c_3 = 0$, because $\psi_3 = 0$ where $\eta_3 = 0$. For regions 2 and 1,

$$f\psi = g'D^2/2 + c_2; \quad f\psi_1 = g'h_1^2 + c_1,$$  

(4.10)

where $c_1$ and $c_2$ are not necessarily zero because $\psi_1$ and $\psi$ may not vanish where $h_1$ and $D$ do. However, since the transport through DC must equal the sum of the transports through EF and AB we have

$$\psi_1 |_{y \to -\infty} = \psi |_{y \to -\infty}.$$  

(4.11)

In addition, we have

$$h_1 |_{y \to -\infty} = D |_{y \to -\infty} = D_\infty.$$  

(4.12)

because ED is set at $y \to -\infty$. Application of (4.11) and (4.12) to (4.10) illustrates that $c_1$ and $c_2$ must be identical ($c_1 = c_2$).

In view of the above considerations, (4.8) takes the form

$$\int_D (h_1 u_1^2 - c_1) dy$$

$$= \int_E (D^2 u^2 - c_1) dy + \int_A h_3 u_3^2 dy,$$  

(4.13)

which, because of the geometry, reduces to

$$\int_D (h_1 u_1^2) dy = \int_E (D^2 u^2) dy + H_z \int_A u_3^2 dy.$$  

(4.14)

This relationship is almost identical to that derived in Nof and Olson (1983) despite the fact that the conditions in the basin into which the fluid is penetrating are quite different. It states that the momentum flux from the two sides of the gap must be equal.

It is important to realize that we have, in fact, assumed that particles do not make a sharp U-turn as they pass through the gap. This essentially implies that $u$ is always positive. Nof and Im (1985) and Nof and Olson (1983) considered a somewhat similar situation. They showed that a negative flow ($u < 0$) in region 2 implies a sharp U-turn near the left edge of the gap (looking off-shore). Such a turn is associated with a singularity, an infinite pressure on the wall, and a breakdown of (4.13).

5. Scaling and manipulations

To obtain the solution we introduce the following nondimensional variables of order unity:

$$u^* = \frac{u}{(g'D_\infty)^{1/2}}; \quad y^* = \frac{y}{[(g'D_\infty)^{1/2} / f]}$$

$$\eta^* = \frac{\eta}{(g'D_\infty / g)}; \quad H^* = \frac{H_z}{D_\infty}$$

$$\delta^* = \frac{\delta}{(g'D_\infty / g)}; \quad \xi^* = \frac{\xi}{D_\infty},$$  

(5.1)

In terms of (5.1), the Bernoulli invariant along the right wall and the intrusion edge (3.3b) becomes,
Using (3.11) and (3.4), relation (5.3) can be written as
\[
\frac{1}{2} [u^*_{3w} + \psi^*_3 (1 - H^*)]^2 = \frac{(\eta^*_{1w})^2}{2} + \eta^*_{1w} + \delta^* .
\] (5.4)

or
\[
\left( \frac{u^*_{3w}}{2} + \psi^*_3 (1 - H^*) \right)^2 + \frac{(\eta^*_{1w})^2}{2} + \eta^*_{1w} + \delta^*. \] (5.4a)

Similarly, the Bernoulli principle along the left wall (3.3c) can be expressed as
\[
\frac{1}{2} (u^*_{3w})^2 + \eta^*_{3w} = \frac{1}{2} (u^*_{2w})^2 + \eta^*_{2w} + \delta^*. \] (5.5)

Upon substitution of (3.4), (3.5) and (2.1) into (5.5) one finds
\[
\frac{1}{2} (u^*_{3w})^2 + \psi^*_3 u^*_{3w} + \psi^*_3 (1 - H^*) \frac{1 - H^*}{2} = \frac{(\xi^*_{2w})^2}{2} + \xi^*_{2w} + \delta^*. \] (5.6)

or
\[
\xi^*_{1w} = \left[ (\psi^*_3 (1 - H^*))^2 - 25* + 1 \right]^{1/2} - 1, \] (5.6a)

where the condition $\eta^* = \xi^*$ (resulting from the no-flow state in the lower layer) has been used.

Using Macsyma\(^2\), relations (3.11), (3.4), (3.5), (2.1) and (4.1) give
\[
6\gamma^*_3 H^* (u^*_{3w})^2 + 6(\gamma^*_3)^2 H^* u^*_{3w} (1 - H^*) \]
\[
+ 2(\psi^*_3)^2 H^* (H^* - 1)^2 \]
\[
+ (\xi^*_2)^2 (2\xi^*_2 + 3) \]
\[- (\xi^*_1)^2 (2\xi^*_1 + 3) = 0. \] (5.7)

With the aid of the above procedure, the problem has now been reduced to a set of three algebraic equations (5.4), (5.6) and (5.7) with three unknowns, $u^*_3$, $\gamma^*_3$, and $\xi^*_2$. The unknowns must also satisfy the following physical restrictions,

\[ H^* = 1 \leq \xi^*_1 < 0, \quad \gamma^*_3 > 0 \]
\[ H^* = 1 \leq \xi^*_2 < 0, \quad \delta^* > 0 \]
\[ 0 < H^* < 1, \quad \eta^*_{3w} > 0 \]
\[ u^*_3 > 0. \] (5.8)

The first and third conditions reflect the impossibility of negative depths and the last condition corresponds to the impossibility of negative flow in region 3.

Substitution of (5.4a) and (5.6a) into (5.7) gives (with the aid of Macsyma) a complicated single equation for $\gamma^*_3$,
\[
2 \left[ (\gamma^*_3)^2 H^* (H^* + 1) - 2\gamma^*_3 H^* [(\gamma^*_3)^2 H^* \]
\[ + (\xi^*_2)^2 + 2\xi^*_2 + 2\delta^* \right]^1/2 \]
\[ + (\xi^*_2 + 1)^2 \right]^{3/2} - 6\gamma^*_3 H^* [1 + \gamma^*_3 \times (H^* + 1)] [\gamma^*_3 \gamma^*_2 H^* + (\gamma^*_3)^2 \]
\[ + 2\xi^*_2 + 2\delta^* \right]^1/2 + 2(\gamma^*_3)^3 H^* \]
\[ \times [(H^*)^2 + 4H^* + 1] + 3(\gamma^*_3)^2 H^* \]
\[ \times [(H^* + 1) + 6\gamma^*_3 H^* [(\xi^*_2)^2 \]
\[ + 2\xi^*_2 + 2\delta^* + (\xi^*_2 + 1)^3 = 0. \] (5.9)

where $H^*$, $\delta^*$ and $\xi^*_2$ are given. The solution of this relatively complicated equation will be discussed in the next section.

6. Solution

Before addressing the most general solution of (5.9), it is instructive to examine the relatively simple case of small flux through the gap.

a. The weak intrusion case

The solution for this case will be obtained by using a perturbation scheme around the no-flux case (i.e., $\delta^* + \eta^*_{2w} = 0$, or equivalently, no initial pressure gradient on the gate). To do so we introduce the transformation
\[ \epsilon = \eta^*_{2w} + \delta^*, \] (6.1)

where $\epsilon \ll T$ but $\delta^* \sim O(1)$ and $\eta^*_{2w} \sim O(1)$. It is possible to obtain the relevant solution by substituting (6.1) into (5.9) and expanding the variables in a power series of $\epsilon$. The algebra turns out to be incredibly tedious, however, and it is much easier to consider the three original equations (5.4), (5.6) and (5.7).

In terms of $\epsilon$, the three original equations (5.4), (5.6) and (5.7) become
\[
\frac{1}{2} [u^*_3 + \gamma^*_3 (1 - H^*)]^2 = \frac{(\xi^*_1)^2}{2} + \xi^*_1 + \delta^*. \] (6.2)

---

\(^2\) Macsyma is a program that enables one to derive complicated analytical expressions with the aid of a computer (see Symbolics 1985). It makes analytical computations much more powerful and, more importantly, it minimizes the possibility of errors.
\[
\frac{1}{2} (u_{sw}^*)^2 + \gamma_3 u_{3w}^* + \frac{(\gamma_3^* )^2 (1 - H^*)}{2} = \left( \epsilon - \delta^* \right)^2 + \left( \epsilon - \delta^* \right) + \delta^* \tag{6.3}
\]
\[
6\gamma_3^* H^* (u_{sw}^*)^2 + 6(\gamma_3^* )^2 H^* (1 - H^*) u_{3w}^* + 2(\gamma_3^* )^2 H^* (H^* - 1)^2 + (\epsilon - \delta^* )^2 \times [2(\epsilon - \delta^* ) + 3] - (\xi_{1w}^* )^2 (2\xi_{1w}^* ) + 3 = 0. \tag{6.4}
\]
We now introduce the expansions
\[
\gamma_3^* (\delta^*, H^*, \epsilon) = \epsilon \gamma_3^{(1)} (\delta^*, H^*) + \epsilon^2 \gamma_3^{(2)} (\delta^*, H^*) + \cdots
\]
\[
u_{3w}^* (\delta^*, H^*, \epsilon) = \nu_{3w}^{(0)} (\delta^*, H^*) + \epsilon \nu_{3w}^{(1)} (\delta^*, H^*) + \epsilon^2 \nu_{3w}^{(2)} (\delta^*, H^*) + \cdots
\]
\[
\xi_{1w}^* (\delta^*, H^*, \epsilon) = \xi_{1w}^{(0)} (\delta^*, H^*) + \epsilon \xi_{1w}^{(1)} (\delta^*, H^*) + \epsilon^2 \xi_{1w}^{(2)} (\delta^*, H^*) + \cdots \tag{6.5}
\]
Substitution of (6.5) into (6.2)–(6.4) gives the O(1) balance
\[
2\delta^* u_{3w}^{(2)} + (u_{3w}^{(1)})^2 + 2\gamma_3^{(1)} (1 - H^*) u_{3w}^{(1)} + (\gamma_3^{(1)})^2 (H^*)^2
- 2H^* [\delta^* \gamma_3^{(2)} + (\gamma_3^{(1)})^2] + 2\delta^* \gamma_3^{(2)} + (\gamma_3^{(1)})^2 - 2(1 - \delta^*) \xi_{1w}^{(2)} - (\xi_{1w}^{(1)})^2 = 0 \tag{6.12}
\]
\[
2\delta^* u_{3w}^{(2)} + (u_{3w}^{(1)})^2 + 2\gamma_3^{(1)} u_{3w}^{(1)} + (\gamma_3^{(1)})^2 (H^*)^2 - 2\delta^* \gamma_3^{(2)} + (\gamma_3^{(1)})^2 (1 - H^*) + 2\delta^* \gamma_3^{(2)} - 1 = 0 \tag{6.13}
\]
\[
6\delta^* \gamma_3^{(1)} H^* u_{3w}^{(1)} + 2\delta^* (\gamma_3^{(1)})^2 H^* (1 - H^*) + 2(\delta^*)^2 \gamma_3^{(2)} + 2\delta^* (1 - \delta^*) \xi_{1w}^{(2)} + (2\delta^* - 1) [ (\xi_{1w}^{(1)})^2 - 1 ] = 0. \tag{6.14}
\]
Solving (6.12) and (6.13) for \(u_{3w}^{(2)}\) and \(\xi_{1w}^{(1)}\),
\[
\frac{[-(u_{3w}^{(1)})^2 - 2\gamma_3^{(1)} u_{3w}^{(1)} + (\gamma_3^{(1)})^2 H^*]}{2\delta^*} \tag{6.15}
\]
\[
\xi_{1w}^{(1)} = \frac{[2\gamma_3^{(1)} H^* u_{3w}^{(1)} - (\gamma_3^{(1)})^2 (H^*)^2 + [2\delta^* \gamma_3^{(2)} + (\gamma_3^{(1)})^2] H^* + (\gamma_3^{(1)})^2 (1 - H^*)]}{2(\delta^* - 1)} \tag{6.16}
\]
and substitution into (6.14) gives the desired relationship between \(\gamma_3^{(1)}\) and \(\xi_{1w}^{(1)}\),
\[
2\delta^* \gamma_3^{(1)} H^* u_{3w}^{(1)} + \delta^* (\gamma_3^{(1)})^2 H^* (1 - H^*) + (\delta^* - 1) [(\xi_{1w}^{(1)})^2 - 1] = 0. \tag{6.17}
\]
The solution of (6.10), (6.11) and (6.17) is finally found to be
\[
\gamma_3^{(1)} = \frac{2(1 - \delta^*)}{\delta^* (H^* - \delta^* + 1)} \tag{6.18}
\]
\[
u_{3w}^{(1)} = \frac{[H^* (1 - \delta^*) + (\delta^*)^2 - 1]}{\delta^* (H^* - \delta^* + 1)} \tag{6.19}
\]
and the first-order balance
\[
\delta^* [u_{3w}^{(1)} + \gamma_3^{(1)} (1 - H^* )] = (1 - \delta^* \xi_{1w}^{(1)}) \tag{6.7}
\]
\[
\delta^* [u_{3w}^{(1)} + \gamma_3^{(1)}] = (1 - \delta^*) \tag{6.8}
\]
\[
(\delta^*)^2 \gamma_3^{(1)} H^* + \delta^* (1 - \delta^*) \xi_{1w}^{(1)} + \delta^* (\delta^* - 1) = 0. \tag{6.9}
\]
Manipulation of (6.7) and (6.8) gives the first-order solution in terms of \(\gamma_3^{(1)}\),
\[
u_{3w}^{(1)} = \frac{[1 - \delta^* - \delta^* \gamma_3^{(1)}]}{\delta^*} \tag{6.10}
\]
\[
\xi_{1w}^{(1)} = \frac{[\delta^* \gamma_3^{(1)} + (\delta^* - 1)]}{(\delta^* - 1)}. \tag{6.11}
\]
It remains to find the first-order solution for \(\gamma_3^{(1)}\) by substituting (6.10)–(6.11) into (6.9). Surprisingly, however, one finds that such a substitution yields an identity. Hence, to close the problem, it is necessary to go to the higher, second-order balances
\[
\xi_{1w}^{(1)} = \frac{-(H^* + \delta^* - 1)}{(H^* - \delta^* + 1)}. \tag{6.20}
\]
The complete nondimensional solution can now be written as
\[
\gamma_3^{(1)} = \frac{2(1 - \delta^*)}{\delta^* (H^* - \delta^* + 1)} + O(\epsilon^2) + \cdots \tag{6.21a}
\]
\[
u_{3w}^{*} = \delta^* + \frac{[H^* (1 - \delta^*) + (\delta^*)^2 - 1]}{\delta^* (H^* - \delta^* + 1)} + O(\epsilon^2) + \cdots \tag{6.21b}
\]
\[
\xi_{1w}^{*} = -\delta^* - \frac{(H^* + \delta^* - 1)}{(H^* - \delta^* + 1)} + O(\epsilon^2) + \cdots \tag{6.21c}
\]
The solution is displayed in Fig. 7.

The onset of upwelling occurs when the upstream interface strikes the shelf break (i.e., \(g_{2w} = H^* - 1\) as shown in Fig. 4) which, for our perturbation solution, turns out to be
\[
\delta^* \geq (1 - H^*) + O(\epsilon^2) + \cdots. \tag{6.22}
\]
special range of parameters and, consequently, does not provide the most general information regarding the onset of upwelling.

The general solution of the polynomial (5.9) can be obtained graphically by plotting the left-hand side as a function of $\gamma_3^*$ and examining the intersections with the axis. One finds that there is always only one root satisfying the constraints given by (5.8). This root (i.e., the value of $\gamma_3^*$) and the corresponding values for the intrusion near-wall speed ($u_{sw}^*$) are shown in Figs. 8–10.

![Figure 7](image1)

**Fig. 7.** The intrusion width $\gamma_3^*$ (upper panel) and the upstream near-wall displacement $\xi_{sw}^*$ (lower panel) as a function of the sea-level difference, $\delta^*$, and the inverse thermocline depth, $H^*$, according to the perturbed solution. The onset of upwelling is shown by the dashed line; the upwelling regime is shaded. The area below the line $u_{sw}^* = 0$ corresponds to an unknown solution that is *unsteady* (hatched region).

This imposes an additional constraint on the perturbed solution in the sense that, if we require that the onset of upwelling will be a part of the solution, then the basic state must be by itself fairly close to upwelling; namely, the interface must be close to the shelf break. The vertical displacement that particles on the interface experience from the initial state to the final one ($d^*$) is $1 + \xi_{sw}^* - H^*$ which, for the onset of upwelling, gives $d^* = \epsilon$.

**b. Graphical solution for the most general case**

Although the perturbed solution presented in the previous subsection is instructive, it corresponds to a

![Figure 8](image2)

**Fig. 8.** The dependence of the intrusion's width $\gamma_3$ (upper panel) and the intrusion's near wall speed $u_{sw}^*$ (lower panel) on the initial sea-level difference $\delta^*$ and $H^*$ (the inverse thermocline depth, $H_*/D_0$) according to the graphical solution. The onset of upwelling is shown by the dashed line. Shaded area denotes upwelling. The area above the line $u_{sw}^* = 0$ and below the line $\gamma_3^* = 0$ corresponds to an unknown solution that is *unsteady* (hatched region).
7. Discussion

Several points should be made before considering the application of our model to the Great Barrier Reef.

a. The infinite thermocline depth limit ($\mathbf{H}^* \rightarrow 0$)

When the depth of the oceanic thermocline approaches infinity, our steady solution breaks down because the intrusion's momentum flux becomes negligible so that the flow-force equation is redundant. In order to obtain the solution for this case, one needs to solve the complete time-dependent problem. Note that

For a fixed ratio between the shelf and thermocline depth, $\mathbf{H}^* = \mathbf{H}_s / \mathbf{D}_o$, the width of the intrusion and the transport of light oceanic water (Fig. 11) into the lagoon increases with increasing the sea-level difference. However, the opposite is true for the oceanic preexisting current—the larger $\mathbf{U}_w^*$ the smaller the intrusion width and transport. Note that the apparent behavioral difference between the graphical solution (Figs. 8–10) and the perturbed solution (Fig. 7) stems from the fact that the former are plotted for constant $\xi_{2w}^*$ whereas the latter is shown for varying $\xi_{2w}^*$ (i.e., constant $\epsilon$). For small $\epsilon$ the two solutions were carefully compared and found to agree [to $\mathcal{O}(\epsilon^2)$] as required.

FIG. 9. The dependence of the intrusion near wall speed $\mathbf{U}_{w}^*$ (upper panel), and the intrusion width $\gamma_3^*$ (lower panel) on the initial oceanic speed $\mathbf{U}_o^*$ and $\mathbf{H}^*$ (the inverse thermocline depth, $\mathbf{H}_t / \mathbf{D}_o$) according to the graphical solution. Dashed lines show the onset of upwelling. Shaded area denotes upwelling.

FIG. 10. The dependence of the intrusion's width $\gamma_3$ (upper panel) and the intrusion's near-wall speed $\mathbf{U}_{w}^*$ (lower panel) on the initial sea level between the ocean and the lagoon ($\mathbf{H}^*$), and the initial near-wall oceanic current $\mathbf{U}_o^*$ according to the graphical solution. The dashed line $\mathbf{H}_{1w}^* = \mathbf{H}^* - 1$ denotes the onset of upwelling (shaded area).
b. The no-initial current limit ($U^* \to 0$)

As in the infinitely deep thermocline case, our steady solution breaks down in this limit. This is due to the fact that it is impossible to balance the integrated mo-

---

**Fig. 11.** The transport of light oceanic water into the lagoon $T_2^*$ (upper panel) and the adjusted upstream transport $T_2^*$ (lower panel) as a function of $\delta^*$ (the sea-level difference between the ocean and the lagoon) and $H^*$ (the inverse thermocline depth, $H/D_w$) according to the graphical solution. The transport has been nondimensionalized by $g'D_w^2/f$; for the initial transport an unrounded value of 0.32 was chosen because it corresponds to the value of $U^*$ considered previously (Fig. 8). As before, the onset of upwelling is shown by the dashed line and the shaded area corresponds to the upwelling region.

---

this cannot be easily verified from Figs. 7–11 which give the mistaken impression that the limit $H^* \to 0$ corresponds to a physically meaningful answer for $\gamma_3^*$, $u_3^*$ and $\eta_3^*$. This is because the variables were scaled with the thermocline undisturbed depth $D_w$; one needs to rescale the variables in order to examine the $H^* \to \infty$ limit.

---

**Fig. 12.** (a) As in Fig. 4 but for $b > \delta_u$. The detailed solution for this case is beyond the scope of this study. (b) The upstream thermocline displacement ($\xi_{uw}^*$) as a function of $\delta^*$ and $U^*$ according to the graphical solution. Results are shown for $H^* = H/D_w = 0.5$ which is appropriate for the Great Barrier Reef. For clarity, the line describing the onset of upwelling (dashed line) was extrapolated for a short distance beyond the regime where it is valid (dashed–dotted line). Regions corresponding to unsteady solutions are hatched.
momentum without a current in region 2. Note that similar behavior was found in the Nof and Olson (1983) study; again, it is necessary to deal with the complete problem in order to solve for this case.

c. The upwelling regime

As mentioned, when \( \delta > \delta_u \), deep bottom water is flowing into the lagoon in the form of a boundary current [Fig. 5 (lower panel) and Fig. 12a]. One would think that this problem could also be solved in detail with the aid of the method used previously for \( \delta \leq \delta_u \). This turned out not to be the case because, no matter how many layers there are on the shelf, there is always only one constraint associated with the flow-force. Consequently, one needs a closure condition and the treatment is much more difficult than it appears to be. In view of this, it is left as a subject for future investigation.

d. Upwelling distance

The vertical distance that particles (along the intersection of the interface with the wall) rise during the time elapsed from the initial state (prior to the lifting of the gate) to the final adjusted state is

\[
d_u^* = 1 + \xi_2^* - H^*. \tag{7.1}
\]

For the onset of upwelling, (7.1) gives

\[d_u^* = \xi_2^* - \xi_1^*,\]

which, as we shall see, can be easily determined from Fig. 12b.

e. The narrow gap limit

Obviously, when the gap’s width goes to zero there is no transport through the gap. The behavior of our solution in this limit cannot be examined because our solution breaks down long before the narrow gap range is reached. When the width becomes much smaller than the Rossby radius, hydraulic jumps and other phenomena involving energy loss may occur so that the Bernoulli integral is no longer valid.

8. Application to the Great Barrier Reef

Before proceeding and calculating the parameters corresponding to the upwelling depth, it is appropriate to comment on the general applicability of our model to the Great Barrier Reef. First, recall that the model does not involve the manner in which the initial current is formed nor does it involve the manner that \( \delta \), the sea-level difference between the two basins, is set up. The current can be a result of various processes, such as wind action, advection or heating. The sea-level difference may result from low atmospheric pressure, wind setup or large-scale variations in the oceanic circulation such as the temporal changes in the East Australian current. Whatever these processes are, they do not directly affect the nature of the upwelling and the flow through the passages.

As far as the geometry of the region is concerned, the chain of coral is, obviously, not a solid wall and there is, no doubt, some flow through the porous chain. Furthermore, during high tide there is some flow above the chain because the sea level is higher \( O (10 \text{ cm}) \) than the coral. The coral is so dense, however, that these flows are probably small compared to the flow through the passages. In view of this, the assumption of a solid wall is probably adequate as a first approximation.

An additional point that should be made in this context is that the neglect of friction may only be partially justified. The lagoon includes many isolated coral heads (“bommies”) that probably slow down the flow. Also, the Ekman number [based on a vertical eddy diffusivity of \( O (10 \text{ cm}^2 \text{ s}^{-1}) \) and a Coriolis parameter of \( 4 \times 10^{-5} \text{ s}^{-1} \)] is of order unity indicating that friction cannot be neglected a priori. While the effect of friction and viscosity can indeed be not entirely negligible in flows through reefs (see Middleton 1983; Middleton and Bode 1988), the inertia terms play a key role in the flow through the gaps. Consequently, one should first understand the inviscid behavior and then proceed to the more complicated frictional flows.

For the purpose of applying our model to the Reef and calculating the transports, it is necessary to determine the thermocline undisturbed depth \( D_\infty \), the lagoon depth \( H_L \), and the deformation radius. The observations of Thompson and Golding (1981) and Grif- fin and Middleton (1986) suggest that, typically, \( D_\infty \sim 100 \text{ m} \) and \( H_L \sim 50 \text{ m} \), and that the density difference between the two layers (\( \Delta \rho/\rho \)) is approximately \( 1.5 \times 10^{-3} \). Hence, for \( f \sim 4 \times 10^{-5} \text{ s}^{-1} \), the deformation radius (based on the thermocline depth) is about 30 km. The long-wave speed \( (g/D_\infty)^{1/2} \) is approximately \( 1.2 \text{ m s}^{-1} \).

It is clear from Fig. 12b that, for \( H^* = 0.5 \), any \( \delta^* > 1/2 \) will produce upwelling in the sense that deep water will be geostrophically sucked from below the thermocline. Such a nondimensional value of \( \delta^* \) corresponds to a dimensional value of about 8 cm. Obviously, many circumstances can lead to such a sea-level difference so that an upwelling is clearly possible. While this is encouraging, it is impossible to quantitatively apply our model to the ocean for two reasons. First, as can be seen from Fig. 12b, the likelihood that an actual physical situation will correspond to the relatively small triangle where there are known steady solutions is very small. It is much more likely that the actual situation will correspond to the areas outside the triangle which, unfortunately, we do not know much about. Second, there is very little information on the statistics of the actual southward flowing oceanic current so that one does not know which value of \( U^*_w \) should be taken, although values of up to \( 1 \text{ m s}^{-1} \) are to be expected.
Fig. 13. A satellite image (courtesy of D. Jupp, CSIRO) of the chlorophyll concentration in the vicinity of a group of passages near Lizard Island in the Great Barrier Reef for 14 April 1981 (see Fig. 1). Note that high concentrations (yellow) are found to the south of the gaps as suggested by our upwelling mechanism (Figs. 5 and 12). Recall that $f < 0$ in this latitude so that Kelvin waves propagate with the wall on their left.
To give the reader a point of reference, it is remarked that point A (Fig. 12b) corresponds to a $U^*_w$ of 0.25 which gives an initially high speed of 30 cm s$^{-1}$. The associated transports in regions 1, 2 and 3 are also high—$1.4 \times 10^6$ m$^3$ s$^{-1}$ and $0.8 \times 10^6$ m$^3$ s$^{-1}$. Also, the initial (i.e., prior to the lifting of the gate) vertical displacement of the interface is 25 m, so that water near the wall has been upwelled a distance of about 25 m. The above transports and speeds may be too high for the Reef because the value of $U^*_w$ that we chose is moderate. For the same reason, the vertical uplifting is relatively small.

In reality, the Reef probably falls into the area below point A where the speeds are much lower and the vertical lift is much higher. As mentioned, the solution for this domain is unsteady and cannot be easily obtained. Despite this aspect, however, it is expected that the line corresponding to the onset of upwelling (dashed line, Fig. 12b) can be extrapolated for a short distance (dashed-dotted line) indicating that the onset of upwelling value of $\delta^*$ is probably adequate. This conclusion is supported by the high chlorophyll concentration shown in Fig. 13, which suggests high nutrient concentration and upwelling similar to that predicted by our model. Note that the location of upwelled water to the south of the gaps cannot be explained by advection within the lagoon because the flow in the lagoon is usually northward rather than southward (Frith, Leis and Goldman 1986).

It is important to emphasize the different roles played by the two upwelling mechanisms. Bernoulli suction is most effective during spring tides when the diurnal and semidiurnal tidal currents are strongest in the passages. Nutrient rich water resulting from such brief transient tidal jets will be symmetrical with respect to north and south and will probably appear as a sequence of partially connected blobs. Any asymmetries can only result from processes that are not directly related to the Bernoulli suction. (For instance, the trade winds may force the flow northward from March to November.) In contrast, geostrophic pumping will be mostly apparent during neap tides when the currents are weak and the sea-level difference can be maintained for several days, or when particular atmospheric conditions somehow cause a sea-level difference between the ocean and the lagoon. Such an upwelling is asymmetrical; it flows southward and is constrained against the outer reef due to the rotation of the earth. It appears that in the Barrier Reef case the geostrophic pumping is a result of neap tides rather than some particular atmospheric conditions. As pointed out earlier (end of subsection 1a), sea-level fluctuations due to neap tide were observed in the area in question. This is consistent with the fact that the image in Fig. 13 was taken on 15 April 1981, and that the previous spring and neap tides occurred on 4 and 11 April, respectively. Namely, the concentration of chlorophyll is hypothesized as entering the lagoon in the week or so prior to the taking of the image, when the neap tide regime held.

Finally, we note that the flux of upwelled bottom water is estimated to be of the same order as the upper water forced into the lagoon, $O(H_0(g\delta)^{1/2}(g^*D_\alpha)^{1/2}/f)$. This scaling is consistent with our previous scalings; it takes into account an intrusion width of the Rossby radius and an intrusion speed corresponding to a sea-level difference of $\delta$.

9. Summary

Prior to giving our conclusions, it is appropriate to mention again the limitations involved in our analysis. The most important assumptions that have been employed are (i) that the coral chains can be approximated by infinitely long walls, (ii) that the flow into the lagoon results from a sea-level difference between the basins, and (iii) that the flow is steady, frictionless, hydrostatic and nondiffusive.

The results of the study can be summarized as follows:

(i) The steady inflow from a light oceanic boundary current into a lagoon (separated from the ocean by long thin walls containing a gap) consists of: geostrophic boundary current, "upstream influence," and a highly nonlinear region in the gap and its immediate vicinity.

(ii) Downstream in the lagoon the inflow is deflected to the right (looking downstream in the Northern Hemisphere). Consequently, this inflow forms a current that is bounded by a wall on the right and a vortex sheet (i.e., a potential vorticity front and a velocity front) on the left (Fig. 3).

(iii) The steady transport, width and speed of the penetrating inflow as well as the upstream alterations have been computed analytically (Figs. 7–11).

(iv) When the sea-level difference ($\delta$) reaches a critical value ($\delta_c$), the interface of the oceanic thermocline strikes the shelf break so that the bottom water reaches all the way to the bottom of the lagoon (Figs. 4 and 5). A further increase in the sea-level difference (i.e., $\delta > \delta_c$) causes an upwelling of bottom water into the lagoon [Fig. 5 (lower panel) and Fig. 12a].

(v) The steady solutions associated with the pre-upwelling states and the onset of upwelling were computed in detail but the state corresponding to upwelling is beyond the scope of the present study. The latter involves a complicated closure condition that requires a lengthy and detailed investigation.

(vi) In analogy to the so-called Bernoulli suction, we refer to the above upwelling mechanism as "geostrophic suction" because it is the geostrophic flow upstream and downstream of the gap that imposes the upwelling. In contrast to the upwelling associated with the Bernoulli suction, the present mechanism does not require any convergence due to funneling. Furthermore, since the fluxes are independent of the gap's width, they are also independent of the number of gaps contained in the coral chain (provided, of course, that
the gap’s width is not very small compared to the Rossby radius).

Qualitative application of the above theory to the Great Barrier Reef suggests that \([O \ (10 \ cm)]\) sea-level differences between the Coral Sea and the adjacent lagoons will cause geostrophic suction and upwelling. Also, our new model and mechanisms suggest that upwelled water will be found to the south of the gaps as has been observed (Fig. 13).

Acknowledgments. This study began when the first author (D. Nof) visited the University of New South Wales (UNSW) as a part of a sabbatical leave during 1987; the support and hospitality of the UNSW is very much appreciated. Travel and support at Florida State University were provided by the Office of Naval Research (Contract N00014-87-J-1209) and the National Science Foundation (grant number OCE-871103). Partial support was also provided by the Australian Marine Sciences and Technology (Grant 851994). Discussions with Rory Thompson regarding the possible frictional effects were very helpful. Enlightening conversations with R. Iverson, R. E. Thomson, Steve Smith and A. Froelich are also acknowledged. Computations and derivations using the “Macsyma” were made by Stephen VanGorder, whose expertise is indispensible.

REFERENCES


