Fission of Single and Multiple Eddies

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ABSTRACT

The analytical results for the splitting conditions of isolated barotropic eddies and the associated final equilibrium state are extended to: 1) nonlinear baroclinic eddies; 2) a group of four nonlinear closely packed eddies, two of which are cyclonic and two of which are anticyclonic (i.e., multiple eddies); and 3) joint nonlinear eddies (i.e., a system consisting of two eddies situated one above the other). The final equilibrium state associated with the group (of four) fission is related to a nonlinear version of geostrophic turbulence and, therefore, is referred to as *ageostrophic turbulence*.

Taking into account that inviscid fission may involve loss of energy via waves radiation, the breakup process is examined by conserving integrated angular momentum, potential vorticity, and mass. The analytical expressions for the conservation of these three properties provide a set of algebraic equations that are solved numerically.

For baroclinic eddies embedded in an infinitely deep lower layer, it is found that, as in the barotropic case, only intense cyclones can break up. This results from the fact that, despite the large amplitude of the nonlinear baroclinic eddies, the offspring are still forced a considerable distance away from their original prebirth center of rotation as is the case with the barotropic eddies. This causes a large gain in angular momentum implying that only eddies whose angular momentum is relatively large to begin with are capable of being potential parents. Again, as in the barotropic case, it turns out that only intense cyclones have large enough angular momentum to allow splitting (because the cyclonic orbital speed is in the same direction as the earth's rotation).

In ageostrophic turbulence, the cyclones break up and the anticyclones merge. Namely, the fission of the cyclones provides the energy necessary for the fusion of the anticyclones. Hence, the final result is a nonlinear system resembling a "Mickey Mouse" (with one large anticyclone and four small cyclones) whose total energy is identical to the total initial energy prior to the fission.

The impossibility of baroclinic anticyclones to break up appears initially to be in contradiction with classical laboratory experiments which show what seems to be an anticyclonic fission. The solution for joint eddies consisting of a cyclone situated above an anticyclone suggests, however, that what really breaks up in the laboratory is the cyclone on top rather than the anticyclone underneath. (Recall that the generation of anticyclones in the laboratory is often accompanied by the formation of a cyclone on top due to convergence.)

It is suggested that the impossibility of baroclinic anticyclones to break up and the tendency of cyclones to split may provide an explanation for the relative abundance of anticyclones in the ocean.

1. Introduction

The fission of isolated eddies is an intriguing subject because of the cyclone–anticyclone asymmetry. Using an application of the conservation of integrated angular momentum to barotropic eddies, Nof (1990a) has recently demonstrated that cyclones are capable of breaking into a group of stationary offspring whereas anticyclones are not. In this preliminary study, it was also demonstrated that lenses with zero potential vorticity (i.e., a special kind of baroclinic eddies) cannot split and break up. While being informative, the Nof (1990a) study left a number of important questions unanswered.

First, it is not clear how would these results be altered when one considers high-amplitude baroclinic eddies which, obviously, are much more applicable to the ocean than barotropic eddies. Second, it is not clear what would be the final equilibrium of a group of closely packed nonlinear baroclinic eddies (i.e., multiple eddies) that are allowed to internally interact. This final equilibrium can be thought of as the nonlinear version of the statistical equilibrium associated with geostrophic turbulence and, therefore, will be referred to as *ageostrophic turbulence*.[1] Third, it is not obvious how Nof's results could be reconciled with the classical laboratory observations of Saunders (1973) which appear to show anticyclonic breaking. It has been suggested that it might be the cyclone on top that actually broke up in Saunders' experiment, but it has not been rigorously demonstrated yet that a system of an anticyclone on the bottom with a cyclone on top (joint

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[1] Note: This name is adopted because of the depth dependency which makes terms such as "two- (or three-) dimensional turbulence on an f-plane" inappropriate.

eddies) can indeed break up. All of the above issues are of significant importance because of their almost direct applicability to the ocean.

The purpose of this paper is to provide at least partial answers to the above questions. Hopefully, this will shed more light on actual fission processes in the ocean. To do so, this paper shall examine the conditions required for the nonlinear breakup of an idealized eddy (Fig. 1), multiple eddies or joint eddies, to a set of stationary eddies adjacent to each other. Considered will be initial baroclinic eddies that consist of a region with uniform potential vorticity corresponding to an anticyclonic or cyclonic circulation. As in Nof (1990a), the vortices are bounded by a free streamline (or, more precisely, a vortex sheet) beyond which the ocean is stagnant, i.e., in contrast to the infinitely large classical point vortices whose orbital speed falls off as $1/r$, the eddies under discussion have a *finite* area. The techniques that have been adapted here are similar to those

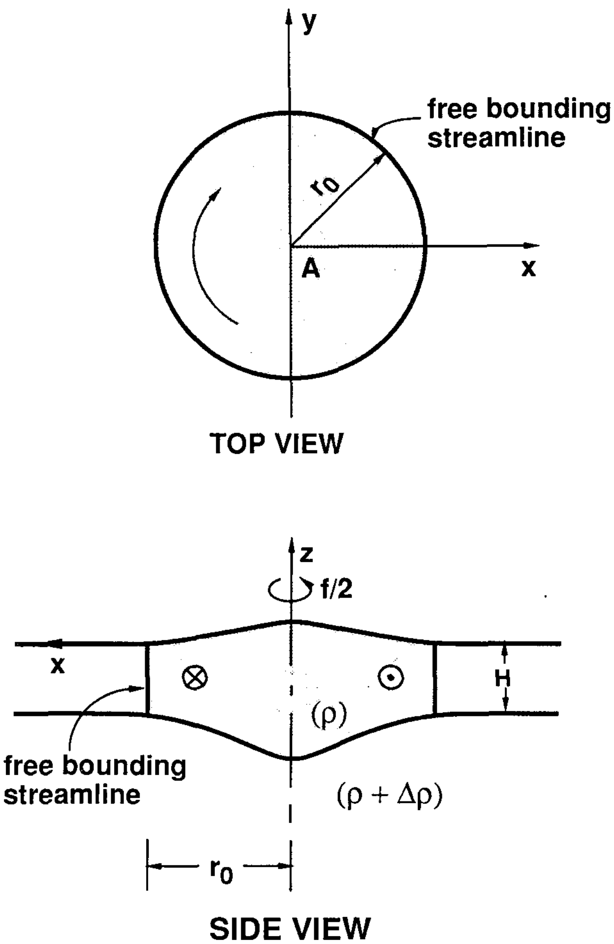

FIG. 1. Schematic diagram of the initial baroclinic vortex. Prior to the splitting, the round vortex is stationary and is bounded by a free streamline (or, more precisely, a vortex sheet) beyond which the fluid is stagnant. As a result of the breakup the eddy is divided into two, three, or four offspring (Fig. 2).

discussed by Nof (1990a) for the barotropic eddies. There is some (but limited) overlapping between the two articles because an attempt has been made to make the present paper self-contained. The essence of the Nof (1990a) study is as follows.

Using a rarely used constraint, the conservation of angular momentum integrated over a *patch,* the study demonstrated that in a *barotropic* system only cyclonic eddies meet the necessary conditions for splitting. Anticyclones can never split, no matter what their structure is. The cyclones are subject to a critical intensity above which breaking up is possible and below which breaking is impossible. This peculiar asymmetry between anticyclones and cyclones results from the fact that the newly formed offspring eddies are pushed away from their original prebirth center of rotation (see Fig. 2, which displays the equivalent baroclinic case) acquiring planetary torque. Therefore, as a consequence of the integrated angular momentum conservation, the torque of the parent eddy must be large enough to accommodate for this addition of planetary torque. It turns out that only cyclones, which typically have more absolute angular momentum than their anticyclonic counterpart (because they rotate in the same sense as the spin of the earth), have enough torque to allow splitting. It shall be shown in the present study that *baroclinic* eddies also have such a parity bias.

The transient splitting problem as a whole is highly nonlinear because both the amplitudes and the Rossby number are of order unity. However, this nonlinearity is not the only difficult aspect of the problem. Free bounding streamlines are notorious because their position and shape are not known in advance, but rather must be determined as part of the problem. Because of these difficulties an attempt to find all the details of the transient splitting process will not be made. Rather, this study will use an integrated angular momentum constraint and other familiar constraints to connect the initial and final states without computing all the details of the transient breakup. Due to the complexity of the associated integrals, the solution of the final algebraic equations is obtained numerically.

The detailed transition between the adopted initial shape and the assumed final state is not of fundamental importance to this solution because only the conditions necessary (but perhaps not sufficient) for the breakup are being sought. The actual final form, which may or may not exist, is, of course, the result of a nonlinear stabilization of the most unstable linear mode. The stability of the eddies with a finite potential vorticity and a finite speed along the edge has not been investigated yet so that it is impossible to say what the details of the time-dependent process would be.

The formulation of the problem and the various scales and constraints are discussed in section 2. Section 3 contains the analysis for the single baroclinic eddies. In section 4 the splitting and the final equilibrium state associated with an initial field of two cyclones and two

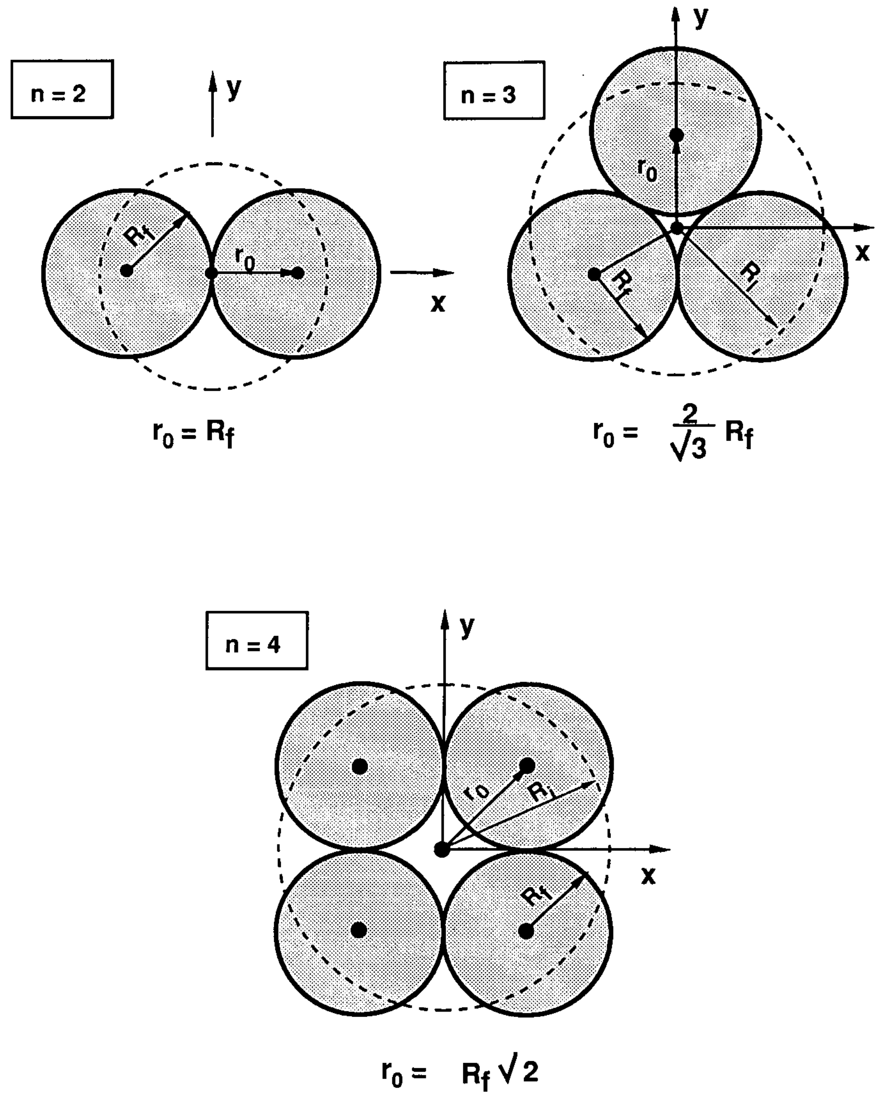

FIG. 2. The assumed final structure of the offspring eddies. Three possible states are shown, n is the number of offspring, R_i is the initial eddy radius, r_0 is the distance from the center of the offspring to the original prebirth center of rotations, and R_f is the final eddies radius. The dashed line shows the edge of the parent eddy (adapted from Nof 1990a).

anticyclones (i.e., multiple eddies associated with ageostrophic turbulence) will be presented. Section 5 addresses the behavior of joint eddies and in section 6 the applicability of the theory to the ocean is discussed. In particular, the relative abundance of anticyclones in the ocean is discussed. Section 7 summarizes this work and points out some of its most important weaknesses.

2. Formulation

a. General description

Consider the parent vortex shown in Fig. 1 and the offspring eddies shown in Fig. 2. Conceptually, the splitting is viewed as being the result of some perturbation on the boundary that grew and subdivided the eddy. It is assumed here that the final state is stationary; otherwise, the physics become inherently time dependent with an energy loss due to radiating Rossby waves in the lower fluid (Flierl 1984), which would prevent the establishment of a steady state. Using not-so-trivial scaling arguments, it will be shortly demonstrated that, for slow splitting processes, the fluid surrounding the vortex can be taken to be *stagnant at all times* (Fig. 3).

b. Governing equations

The "reduced gravity" shallow-water equations for the upper layer (i.e., the interior and exterior shown in Fig. 3) are

$$(\partial u/\partial t) + u(\partial u/\partial x) + v(\partial u/\partial y) - fv + g'(\partial h/\partial x) = 0 \quad (2.1)$$

$$(\partial v/\partial t) + u(\partial v/\partial x) + v(\partial v/\partial y) + fu + g'(\partial h/\partial y) = 0 \quad (2.2)$$

$$(\partial h/\partial t) + (\partial(hu)/\partial x) + (\partial(hv)/\partial y) = 0, \quad (2.3)$$

where the notation is conventional (i.e., u and v are the horizontal velocity components, f the Coriolis parameter, g' the reduced gravity, and h is the depth).

The above equations can be manipulated to give the familiar conservation law for potential vorticity,

$$\frac{D}{Dt}\left(\frac{\partial v/\partial x - \partial u/\partial y + f}{h}\right) = 0 \quad (2.4)$$

and the conservation of energy,

$$J[(u^2 + v^2)/2 + g'\xi; \Psi] + (h/2)[\partial(u^2 + v^2/\partial t)] = 0 \quad (2.5)$$

where J is the Jacobian, Ψ is a streamfunction (defined by $\partial\Psi/\partial y = -uh$; $\partial\Psi/\partial x = vh$), and $h = H + \xi$ (here, H is the undisturbed depth and ξ is the interface displacement which is measured positively downward). For steady processes ($\partial/\partial t = 0$), relation (2.5) reduces to the familiar Bernoulli integral,

$$(u^2 + v^2)/2 + g'\xi = B(\Psi). \quad (2.6)$$

Equations (2.1)–(2.3) can also be used to derive the conservation of integrated angular momentum over a patch of fluid with an area S (see Nof 1990a,b),

$$\frac{d}{dt}\iint_S [xv - yu + f(x^2 + y^2)/2]hdxdy - \underbrace{\frac{g'}{2}\oint h^2(xdx + ydy)}_{\text{(I)}} = 0, \quad (2.7)$$

where the arrowed circle indicates counterclockwise integration around S.

The first term on the left-hand side of (2.7) is the torque associated with the planetary and orbital speed within S, whereas the second is the flux of torque through the boundary of S. In Nof's (1990a) barotropic study, the second term was neglected on the ground that gentle and slow fission processes will not have much exchange of angular momentum with their environment. In what follows it will be demonstrated in a more rigorous manner that for slow splitting processes the exchange of angular momentum between the patch (S) and the surrounding water (I) is indeed small so that the neglect of the second term is justified and the integrated angular momentum can be expressed as,

$$\boxed{\frac{d}{dt}\iint_S [xv - yu + f(x^2 + y^2)/2]hdxdy = 0.} \quad (2.7a)$$

c. Scaling

The scaling associated with the fission is not trivial and, therefore, detailed explanations are given in the following text. The reader who still finds some of these

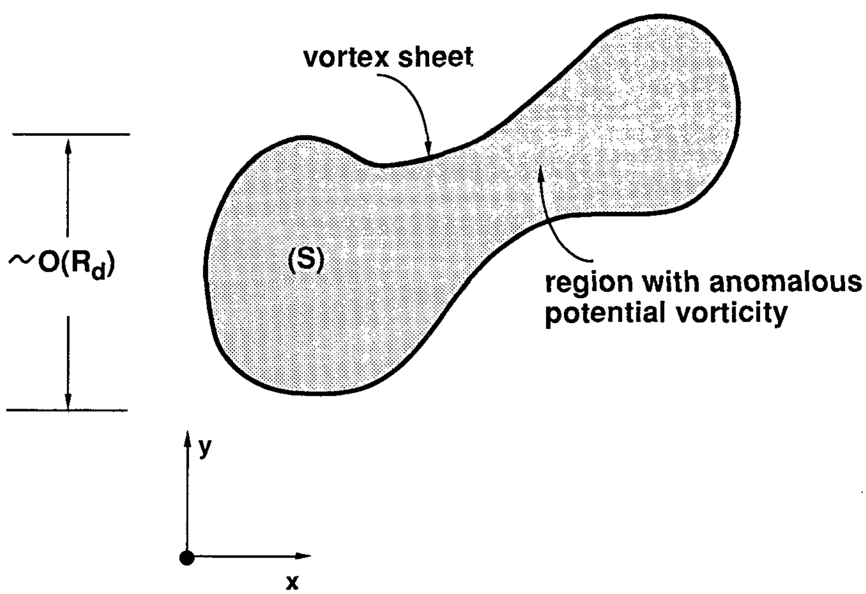

FIG. 3. Schematic diagram of a baroclinic patch with an anomalous potential vorticity. This "peanut-shaped" patch (conceptually) represents the transient processes leading the parent eddy to the offspring eddies.

details to be too complicated is advised to continue reading and return to these points later.

The scaling analysis begins by noting the following points. First, it is noted that the baroclinic eddy length scale is of the order of the deformation radius $R_d \equiv (g'H)^{1/2}/f$ (where H is the undisturbed depth) and that the scale of the orbital speed is $(g'H)^{1/2}$ so that the spinning time scale is $O(f^{-1})$. These scales are typical for nonlinear eddies. Second, it is noted that, since the second integral in (2.7) is taken over a closed contour, only the interface deviation from a state of rest (ξ) contributes to nonzero values. Namely,

$$I \sim O\left[g'\oint H\xi(xdx + ydy)\right]. \quad (2.8)$$

Third, it is noted that the speeds outside the patch are of $O(R_d/T)$ (where T is the splitting time scale) because all the exterior motions result from the movements of the vortex sheet (i.e., the boundary of S).

Leaving these three points aside for a moment, it is noted that in analogy to merging processes most of which have been observed to be slow compared to the circulation within the eddies (e.g., Nof and Simon 1987; Nof 1988) it can be assumed that

$$T \gg (f^{-1}). \quad (2.9)$$

This assumption is also supported by the numerical experiments of Cushman-Roisin and Tang (1990) where both fission and fusion processes were noted to be slow.

For slow splitting processes the boundary of the patch (i.e., the edge of the splitting eddy) can be taken to be a streamline [see(2.5)]. Hence, one finds from (2.6) that *along* the boundary $g'\xi \sim O(u^2) \sim O(R_d/T)^2$. Combination of this information with (2.8) and (2.7) shows that the torque flux (I) can be neglected as long as

$$(R_d/T)^2 R_d^2 \ll fR_d^2(R_d^2/T),$$

which is identical to the original assumption (2.9) regarding the slowness of the splitting process ($T \gg f^{-1}$). It states that the leakage of torque through the boundary is so small that even though it is active over a long period of time it is still negligible. This completes the scaling analysis and shows that for slow fission integral I can indeed be neglected.

d. Constraints

The initial and final states are connected with the aid of the following principles.

(i) Potential vorticity: since the model is inviscid, potential vorticity is conserved so that (in polar coordinates),

$$\boxed{\begin{aligned}&\{r^{-1}[d(rv_{\theta i})/dr] + f\}h_i^{-1}\\ &\quad = \{r^{-1}[d(rv_{\theta f})/dr] + f\}h_f^{-1} = f(H_p)^{-1}\end{aligned}} \quad (2.10)$$

where H_p is the "potential vorticity depth," and, as before, r is the radius. The subscripts "i" and "f" denote the initial and final states.

(ii) Volume conservation: this constraint can be written as

$$\boxed{\iint_{S_i} h_i r dr d\theta = n\iint_{S_f} h_f r dr d\theta} \quad (2.11)$$

where n is the number of offspring.

(iii) Angular momentum: following the earlier analysis;

$$\iint_{S_i} (fr^2/2 + rv_{\theta i})h_i r dr d\theta = n\iint_{S_f} (fr^2/2 + rv_{\theta f})h_f r dr d\theta. \quad (2.12)$$

Next, it is noted that for a vortex with a radius R_f situated a distance r_0 away from its original prebirth center of rotation, the integrated angular momentum (AM) is

$$\text{AM} = 2\pi\int_0^{R_f} [(fr_0^2/2) + fr^2/2 + rv_{\theta f}]h_f r dr,$$

where r and v_θ are now measured relative to the new eddy's center (e.g., Cushman-Roisin 1989; Nof 1990a,b). In view of this, it is found that (2.12) can be expressed as

$$\boxed{\begin{aligned}&\int_0^{R_i} [(fr_0^2/2) + rv_{\theta i}]h_i r dr\\ &\quad = n\int_0^{R_f} [(fr_0^2/2) + fr^2/2 + rv_{\theta f}]h_f r dr.\end{aligned}} \quad (2.13)$$

(iv) Energy: The loss of energy during the fission is

$$E_i - E_f = \iint_{S_i} [h_i v_{\theta i}^2/2 + g'\xi_i^2/2]dxdy - n\iint_{S_f} [h_f v_{\theta f}^2/2 + g'\xi_f^2]dxdy. \quad (2.14)$$

In the following sections, this paper connects the final and initial states with the aid of (2.10), (2.11), and (2.13) verifying that (2.14) gives a positive (or no) loss.

3. Splitting of single baroclinic eddies

Before discussing the solution for the fission of single baroclinic eddies it should be pointed out that, in addition to the constraints mentioned earlier, the interior of both the parent and the offspring obeys the potential vorticity and momentum equations,

$$r^{-1}[d(rv_\theta)/dr] + f = hf(H_p)^{-1} \quad (3.1)$$

$$v_\theta{}^2/r + fv_\theta = g'dh/dr, \quad (3.2)$$

which are subject to the boundary conditions:

$$v_\theta = 0; \quad r = 0 \quad (3.3)$$

$$h = H; \quad r = R. \quad (3.4)$$

These boundary conditions imply that the orbital speed at the center vanishes (3.3) and that the depth is finite along the rim ($r = R$). Note that, since the model is inviscid, the orbital velocity is not necessarily zero along the edge.

The above equations and their boundary conditions have an analytical solution only for the case when $H_p \to \infty$ (i.e., zero potential vorticity lens) whose breakup was examined previously in Nof (1990a). For the most general nonlinear case [$H \sim O(H_p)$, $\xi \sim O(H)$] the set [(3.1)–(3.2)] must be solved numerically (e.g., Csanady 1979; Flierl 1979). This was done using a shooting method that employs a fifth-order Runge–Kutta solver from the Numerical Recipes Software Package (Press et al. 1986). Before continuing, it is worth pointing out that Csanady (1979) has attempted to solve the set [(3.1)–(3.2)] analytically by neglecting the centrifugal acceleration in the momentum equation. Strictly speaking, this can only be done for quasi-geostrophic eddies (i.e., small amplitude) and is not permitted in the nonlinear case (Flierl 1979).

To obtain the splitting conditions the numerical solution of (3.1)–(3.2) (as a function of the unknown potential vorticity depth H_p) is substituted into the two contraints (2.11) and (2.13). Also, for each mathematical configuration (i.e., n = 2, 3, or 4), the appropriate r_0 (as a function of R_f) is used (see Fig. 2). For a given parent radius R_i, this procedure gives a system of two algebraic equations with two unknowns (H_p and the final radius R_f) which are also solved numerically using a modification of the Powell hybrid method from the software package MINPACK. The unique solutions are shown in Figs. 4 and 5 which clearly illustrate that only cyclones can split. Note that values above the "critical" curves in the upper panel of Fig. 4 correspond to offspring eddies that are not kissing but rather are separated from each other. Also, note that the plotted curves correspond to the *only* physically relevant solution (i.e., eddies with positive depth everywhere) regardless of the sign of ($E_i - E_f$). Namely, no other physically relevant solutions are found even if one allows for an energy gain instead of a loss. The main difference between these nonlinear baroclinic conditions and those found earlier by Nof (1990a) is that now the minimum intensity necessary for fission is a function of the parent eddy size (Fig. 4) as opposed to the constant found in the barotropic case [see (his) Fig. 5]. When the size of the baroclinic eddies goes to zero one recovers the barotropic results as should, of course, be the case.

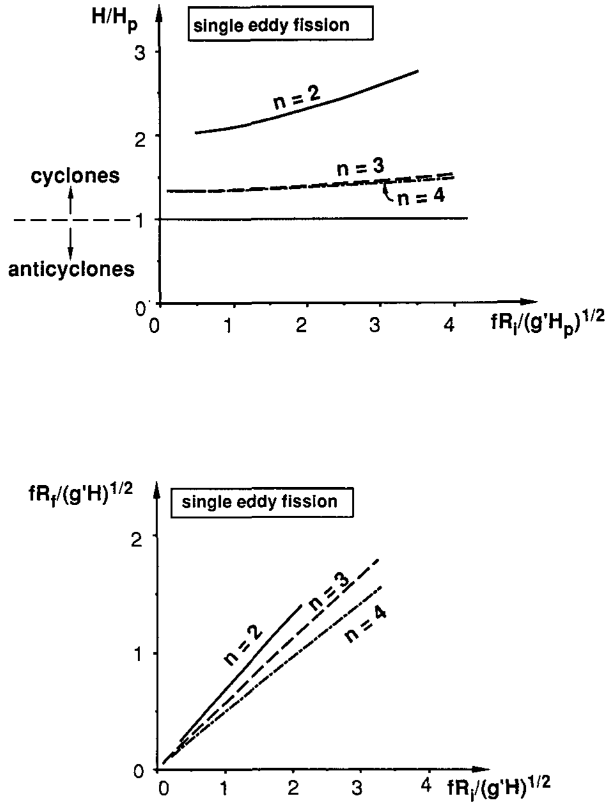

FIG. 4. Top: the minimum ratio between the undisturbed depth (H) and the potential vorticity depth (H_p) necessary for a single eddy fission to occur—as a function of eddy's size. Values of H/H_p that are smaller than unity correspond to anticyclones, whereas values larger than unity correspond to cyclones. Lenses are represented by $H = 0$ and $H_p \neq 0$. In order for fission to occur it is required that H/H_p be above the computed curves, i.e., for splitting into two offspring the potential vorticity of the parent vortex must be above the solid line and for splitting into three it needs to be above the dashed line. This implies that all anticyclones cannot split. Note that the size is scaled with the deformation radius based on the potential vorticity depth H_p rather than the undisturbed depth H, so that the limit $H \to 0$ (i.e., lenses with various potential vorticities) can be taken. Bottom: the offspring radius (R_f) as a function of the parent eddy radius (R_i) for the splitting of a single vortex.

Figure 5 indicates that the energy loss increases monotonically with increasing number of offspring. Furthermore, it can be shown that all the energy is lost when the parent eddy breaks into an infinite number of offspring eddies suggesting a "blue" cascade toward small wavelength. This appears, at first, to be in contradiction to the well-known "red" cascade toward large wavelength (e.g., Rhines 1977). A closer look reveals, however, that this is not the case. The known red cascade is associated with an *eddy field* whereas the blue cascade is associated with a *single* initial eddy. It will be seen later in section 4 that when the interactions and breakup of *several* eddies is considered, the blue cascade is no longer observed.

4. Multiple eddies fission and fusion

In this section it will be demonstrated that the energy released during the fission of cyclones may provide the excess energy required for the *merging* of anticyclones. Namely, using angular momentum conservation it shall be shown that in an interactive field consisting of nonlinear cyclones and anticyclones, (i.e., ageostrophic turbulence), the nonlinear cyclones will tend to break up whereas the nonlinear anticyclones will tend to merge. The fact that this is what usually happens in inviscid models has been demonstrated numerically by Cushman-Roisin and Tang (1990, see their Fig. 5) who performed various computations but did not provide a detailed analysis of the process.

Consider the four adjacent eddies shown in the upper panel of Fig. 6; two of the eddies are cyclonic and two are anticyclonic. The eddies are allowed to interact and the question is whether or not the ageostrophic system can be converted to a final equilibrium state consisting of five eddies as shown in the lower panel of Fig. 6. Under such a fission–fusion conversion, the cyclones break up whereas the anticyclones merge to form a "Mickey Mouse" pattern. In contrast to the previous cases of single and joint eddies where energy was allowed to radiate away, here, the total energy of the system is kept fixed so that energy can only be exchanged within the system itself. For simplicity, first barotropic systems, which can be solved analytically, shall be considered and then the more nonlinear complicated baroclinic analysis whose algebraic constraints must be solved numerically will be considered.

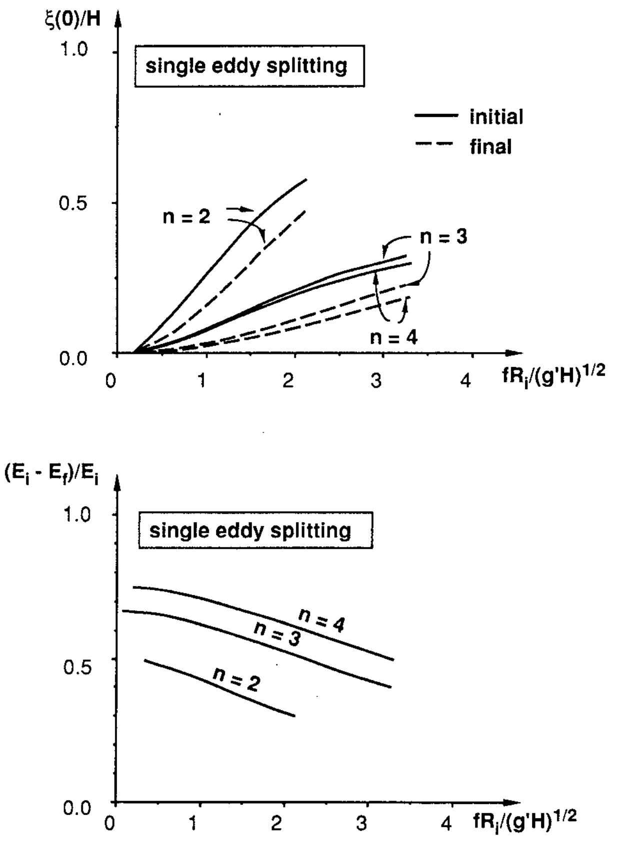

FIG. 5. Top: the central amplitude [$\xi(0)$] of the initial (parent) eddy and the final offspring eddies as a function of the parent eddy size. As expected, the central amplitudes decrease due to the fission. Bottom: the relative energy loss $(E_i - E_f)/E_i$ as a function of the parent eddy size.

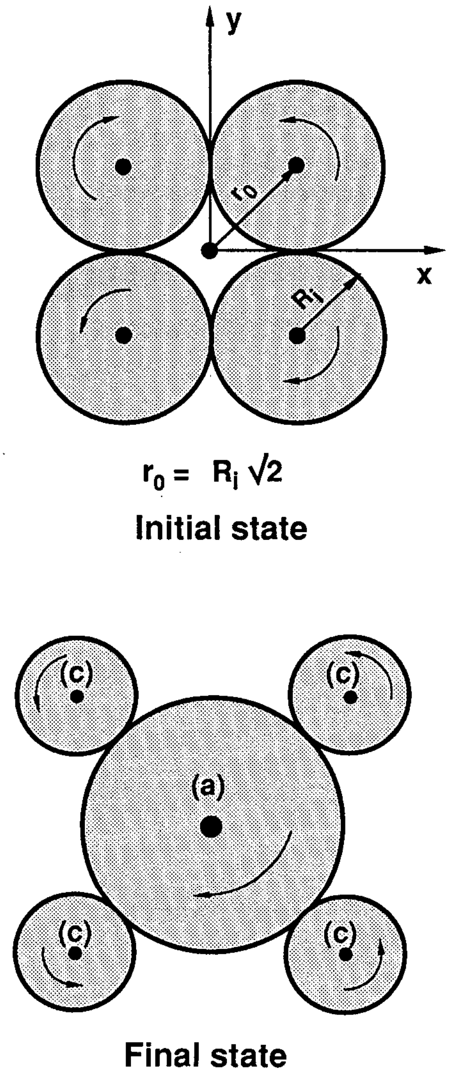

FIG. 6. The initial and final state of the closely packed multiple eddy system. The initial cyclones and anticyclones interact and form a new "Mickey Mouse" pattern which includes one large anticyclone and four small cyclones. The fission of the cyclones provides the excess energy necessary for the merging of the anticyclones (see text).

a. Multiple barotropic eddies

It is assumed here that the free surface vertical displacement (η) is small as far as its relationship to the depth is concerned. However, as in Nof (1990a), it is important as a mechanism for the redistribution of energy.

The vorticities of both the cyclones and anticyclones are taken to be uniform so that the velocity distributions are

$$v_{\theta a} = Afr; \quad v_{\theta c} = Bfr, \tag{4.1}$$

where the subscripts "a" and "c" denote anticyclones and cyclones, respectively. Conservation of mass implies that, since the initial radius of all the eddies is R_i, the final radius of the anticyclone is $R_i\sqrt{2}$ and that of the cyclones is $R_i/\sqrt{2}$.

Conservation of total energy ($\int\int v_\theta^2 ds$) in the system implies,

$$B^2 = 2A^2 \tag{4.2}$$

whereas conservation of total angular momentum gives

$$2B = 4A + 3 \tag{4.3}$$

The system (4.2)–(4.3) has two real roots,

$$\left.\begin{aligned} A_1 &= -0.439 \\ B_1 &= 0.621 \end{aligned}\right\} \tag{4.4}$$

and

$$\left.\begin{aligned} A_2 &= -2.56 \\ B_1 &= -3.62 \end{aligned}\right\}. \tag{4.5}$$

The first pair of roots (4.4) corresponds to an initial system consisting of two cyclones ($B_1 = 0.621$) and two anticyclones ($A_1 = -0.439$), and a final system consisting of four small cyclones and one large anticyclone. This is the solution that was sought. It corresponds to an energy transfer from the cyclones to the anticyclones. The second pair of roots (4.5) corresponds to an initial system consisting of four anticyclones and a final system consisting of five anticyclones. The initial vorticity of these anticyclones of $[(r^{-1}) \times d(rv)/dr + f]$ is large and negative so that, due to inertial instability, they are probably highly unstable and, hence, cannot exist in the first place. In view of this, the second pair of roots is rejected as being unphysical.

The physically relevant roots correspond to "critical" values for the vorticities (points 1 in Fig. 7). Since there can be some energy loss during the fission–fusion process and the final eddies can be separated from each other instead of kissing, it is found that conversion is always possible whenever the intensities fall above the critical point shown in Fig. 7.

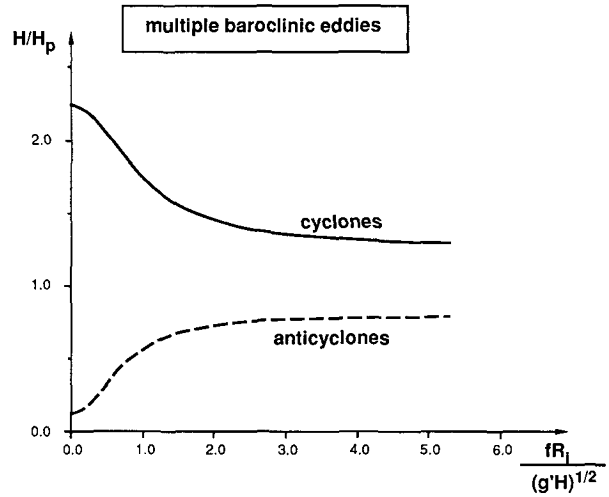

FIG. 8. The ratio between the undisturbed depth and the "critical" potential vorticity depth of the initial cyclones (solid line) or anticyclones (dashed line) as a function of the initial eddies size. In order for conversion (i.e., the splitting of cyclones and merging of anticyclones) to occur, the cyclone's potential vorticity depth must fall within the range shown in Fig. 9.

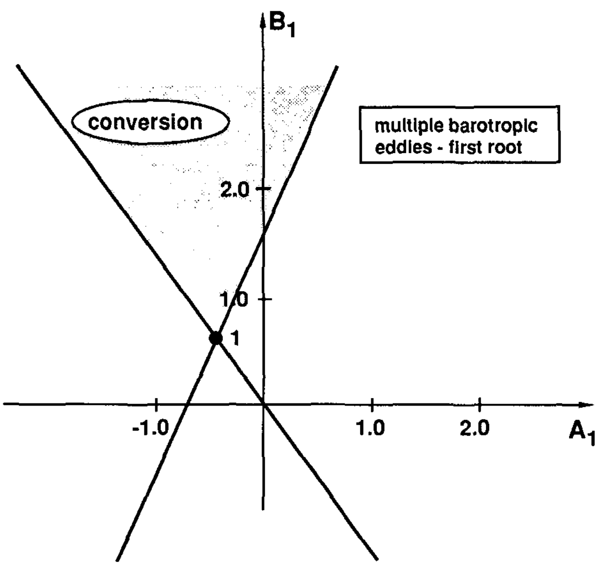

FIG. 7. The "critical" vorticities of the multiple barotropic eddies, i.e., the vorticities corresponding to a fission–fusion conversion with no energy loss and kissing eddies (point 1). The shaded area denotes a region where conversion (with some energy loss and separated eddies) is possible.

b. *Multiple baroclinic eddies*

Here, again consider an initial system of four closely packed eddies and a final system of five eddies as shown in Fig. 6, but now the eddies are overlying an infinitely deep (passive) lower layer. As in the single baroclinic eddies case, the solution for each individual vortex is found numerically and then a set of algebraic equations resulting from the constraints is obtained. Again, this set is solved numerically. The results are shown in Figs. 8–10. The range of potential vorticities associated with a possible conversion is displayed in Figs. 8, 9. The top panel of Fig. 10 shows the size of the offspring whereas the lower panel shows the amplitudes at the center of the eddies. As in the barotropic case, the fission of the cyclones provides the energy necessary for merging of the anticyclones. This is consistent with the well-known difficulty in merging inviscid eddies without an external energy source (Gill and Griffiths 1981; Overman and Zabusky 1982; Griffiths and Hopfinger 1986, 1987; Nof and Simon 1987; Nof 1988; Cushman-Roisin 1989).

5. Joint eddies splitting

The purpose of this section is to demonstrate that the apparent splitting of a lens in the laboratory (e.g., Saunders 1973; Griffiths and Linden 1981) is probably not due to the fission of an isolated anticyclone but rather is the mutual fission of a lens and its conjugate cyclonic vortex on top. This idea was already mentioned in Nof (1990a) but no analysis that could rigorously support it has been presented there. Here, it shall be attempted to put this idea on a firmer ground by actually computing the conditions necessary for the splitting of a joint system.

To do so, consider the system shown in Fig. 11, where the lens is embedded in a cyclone that is bounded by a free streamline. The potential vorticity of the lens on the bottom is zero whereas the cyclone's (uniform) potential vorticity is f/H_p where $H_p \neq H$. Recall that very often lenses are formed in the laboratory by the collapse of a cylinder so that an intense cyclone is always formed on top due to convergence (e.g., Nof and Simon 1987; Dewar and Killworth 1990; Saunders 1973; Griffiths and Linden 1981; Mory et al. 1987; Kostianoy and Zatsepin 1989). The actual cyclone in the laboratory has a potential vorticity identical to that of the environmental fluid (i.e., $H = H_p$) but such a case cannot be solved with these methods. This results from the fact that, for such conditions, there are no means of tracing the upper fluid containing the cyclone because it can freely exchange mass with the surrounding fluid during the splitting. Also, as will be seen, the solution for the cyclones has a vanishing speed at $r = 0$ (Nof and Simon 1987, see also Dewar and Killworth 1990) which cannot easily be handled and, therefore, will not be considered.

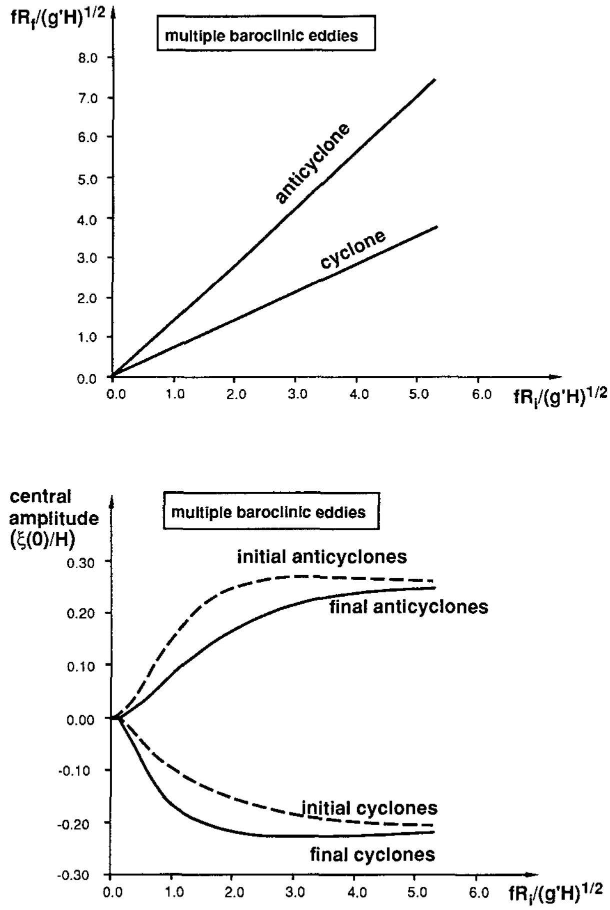

FIG. 10. Upper panel: the final multiple eddies radii as a function of the critical initial size. Lower panel: the initial and final central amplitudes as a function of the initial eddy size.

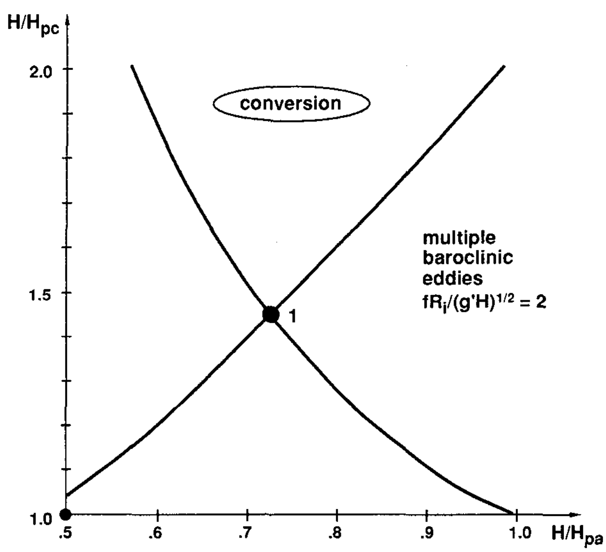

FIG. 9. The regime of the fission–fusion process (for one initial radius). As in the barotropic case, since some energy loss might be present and the final eddies can be separated from each other, conversion is always possible if the variables are above the critical point (point 1). (Recall that the critical point corresponds to no total energy loss and kissing eddies.) The quantities H_{pa} and H_{pc} are the "potential vorticity depth" of the anticyclones and cyclones, respectively.

Both the initial and final lens have the velocity profile,

$$v_{\theta 2} = -fr/2. \tag{5.1}$$

The lens depth is governed by

$$r^{-1}v_{\theta 2}^2 + fv_{\theta 2} = g'(dh/dr) + \rho^{-1}(dp_1/dr), \tag{5.2}$$

where p_1 is the pressure in the upper layer. Equation (4.2) is subject to the boundary condition $h = 0$ at r

$= R_2$. The initial and final upper layer cyclones are governed by the equations,

$$r^{-1}v_{\theta 1}^2 + fv_{\theta 1} = \rho^{-1}(dp_1/dr) \quad (5.3)$$

$$r^{-1}[d(rv_{\theta 1})/dr] + f = h_f/H_p, \quad (5.4)$$

subject to the boundary conditions

$$v_{\theta 1} = 0 \quad r = 0 \quad (5.5)$$

$$h_1 = H \quad r = r_1 \quad (5.6)$$

which imply vanishing orbital speeds at the center and discontinuous speeds along the edge. If, in addition to (5.6), one imposes the condition that the velocities be continuous along the edge, then the first condition (5.5) must be relaxed and replaced by $v_{\theta 1} \to \infty$ at $r = 0$ as is the case with the laboratory cyclone (see Nof and Simon 1987). As before, the system (5.3)–(5.6) does not have an analytical solution (Csanady 1979; Flierl 1979) and, hence, must be solved numerically. As mentioned, because of numerical limitations, only situations where the velocity at $r = 0$ goes to zero will be considered; in view of this, the limit of the solution when $H \to H_p$ will not yield the equivalent laboratory case where the velocities go to infinity as $r \to 0$.

As before, the initial and final states are connected using conservation of potential vorticity, mass, and integrated angular momentum. Note, however, that there are now two mass conservation constraints (one for each layer) but only one integrated angular momentum constraint because angular momentum can easily be exchanged between the lens and the cyclone on top. As in the single eddies splitting analysis, it is assumed here that the splitting process is slow compared to the circulation period so that there is no exchange of angular momentum between the joint eddies and their environment. Also, as before, a set of algebraic equations that are solved numerically is obtained from the various constraints.

The results are shown in Figs. 12 and 13. The upper panel of Fig. 12 illustrates that the joint system can break up whenever the cyclone's intensity is above a critical value. The lower panel of Fig. 12 gives the initial and final radii of the lens and cyclone. Note that, due to the two independent mass conservation constraints, it was possible to choose a case where the radius of the initial lens is smaller than that of the initial cyclone but the final radii are identical. Figure 13 displays the

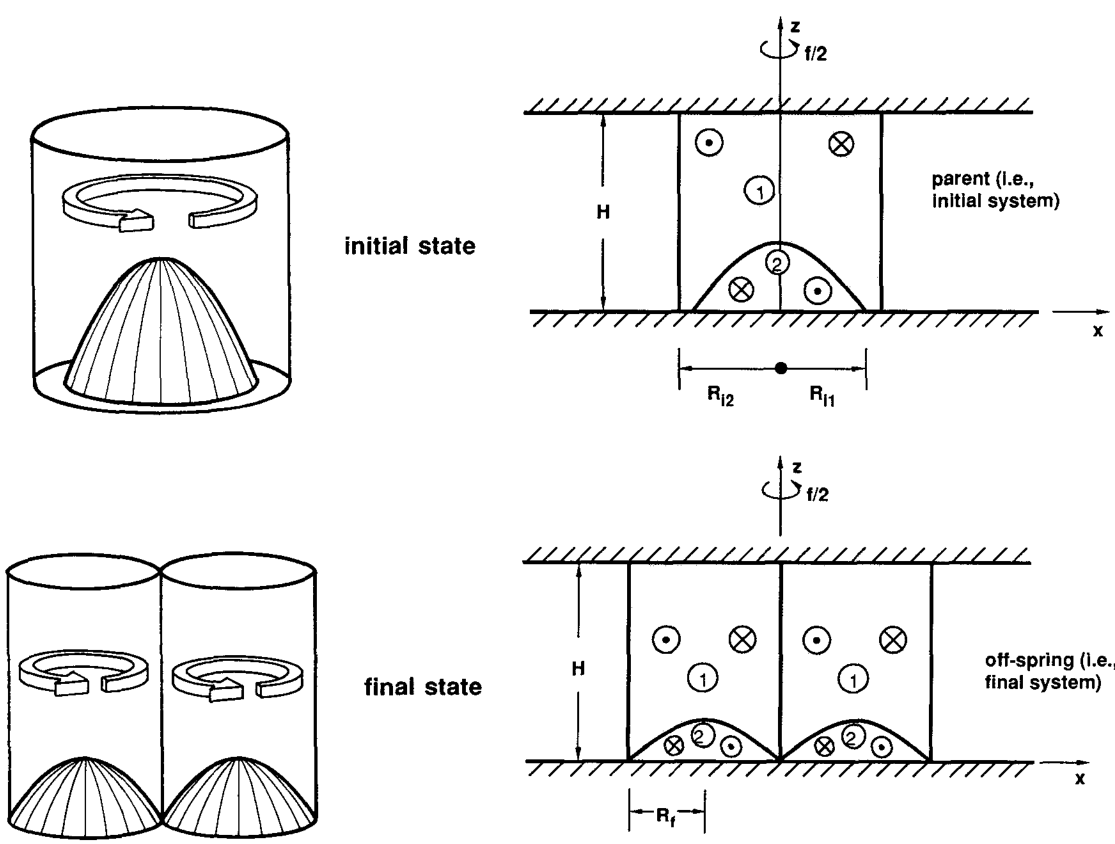

FIG. 11. (a) Schematic three-dimensional view of the joint eddies fission (left panel). The radius of the initial cyclone on top is slightly larger than that of the lens underneath; the difference in the radii is such that in the final state the radii of the lenses and cyclones are identical. (b) Schematic side view of the joint vortices prior to and after the splitting (right panel). The lens on the bottom (i.e., fluid #2) has zero potential vorticity whereas the cyclone on top (fluid #1) has a finite potential vorticity f/H_p.

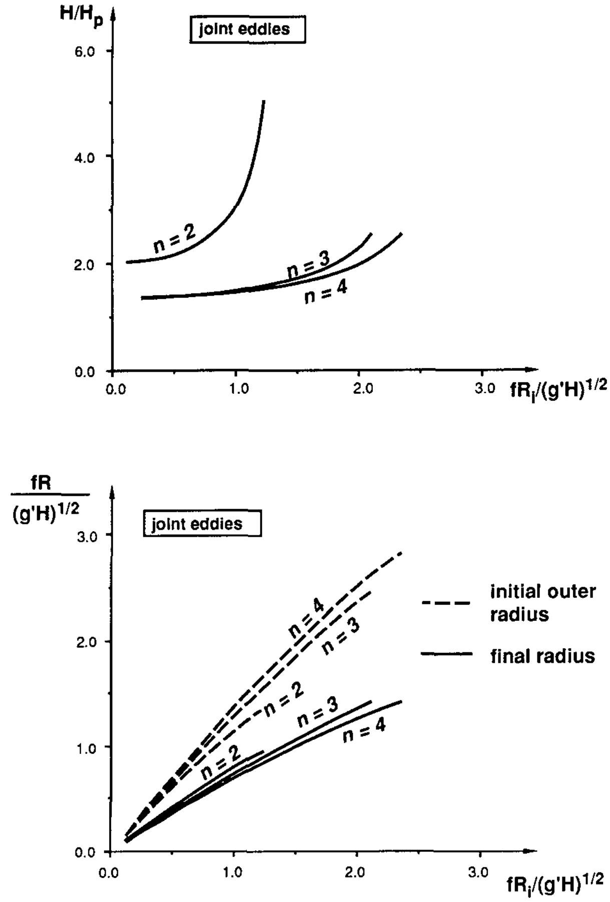

FIG. 12. Top: the "critical" ratio between the undisturbed depth and the potential vorticity depth (of the cyclone on top) required for joint vortices splitting—as a function of the initial eddy size. Bottom: the initial radius of the cyclone (dashed lines) and the final radius of the lens (and cyclone) as a function of the initial lens radius.

lens central depth (upper panel) and the energy loss during the breakup. Although the lens does not reach all the way to the top, its depth is of the same order as that of the surrounding fluid.

The previous analysis illustrates that, even though a lens on its own cannot split, once it is combined with a cyclone there can be sufficient initial angular momentum to permit fission. The suggestion that the laboratory splitting is due to the cyclone is supported by the experiments of Kostianoy and Zatsepin (1989) who observed that lenses formed by an injection, rather than a cylindrical collapse, did not break up. However, on the other hand, the experiments of Griffiths and Linden (1981) show that, in their laboratory setting, even lenses formed by an injection broke up. In view of this, the question of what actually breaks up in the laboratory (lenses or joint eddies) remains, at least partially, open.

6. Applications

This section will discuss the possibility that the relative abundance of anticyclones in the ocean is related to the crucial role of angular momentum in the splitting process and the associated tendency of cyclones to break in ageostrophic turbulence.

The fact that the majority of eddies whose length scale is $\sim O(10 \text{ km})$ are anticyclones has been documented for both mid- and high-latitude oceans (McWilliams 1985). For example, of 98 eddies observed in the Arctic Ocean during the mid-1970s, 95 were anticyclonic (Manley and Hunkins 1985; see also Kostianoy and Belkin 1989). This cyclone–anticyclone asymmetry is not observed in the somewhat larger ($\sim$50 km) eddies such as Gulf Stream rings (i.e., the number of cyclonic rings is almost identical to that of anticyclonic rings). McWilliams (1985) argues that, while these (mesoscale) rings have a length scale comparable to the deformation radius (based on the properties of the surrounding fluid), the smaller anticyclonically biased eddies have a length scale that is smaller than the above radius. He, therefore, suggested that they be referred to as submesoscale vortices. Although McWilliams' classification is useful, when the deformation radius is defined on the basis of the

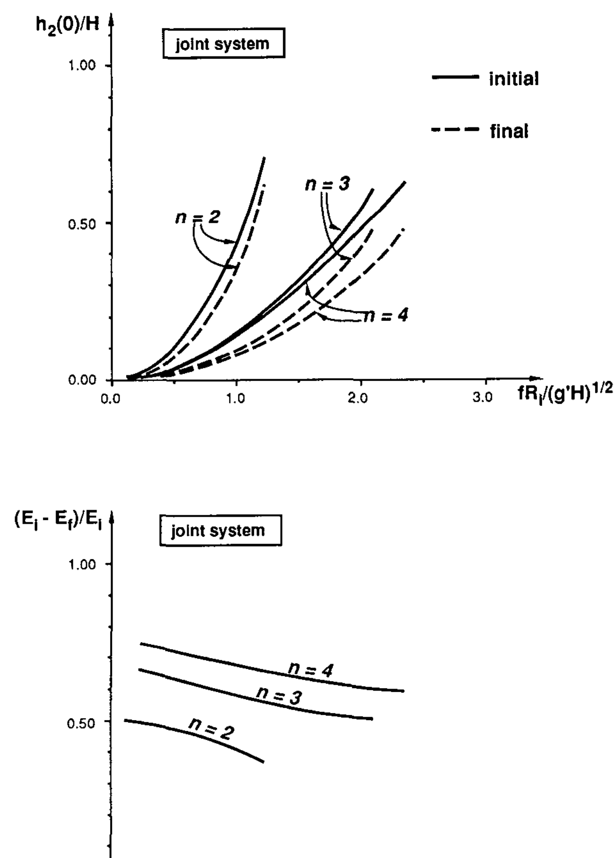

FIG. 13. Upper panel: the lens central depth [$h_2(0)$] prior to and after the joint system fission. Lower panel: the relative energy loss during the fission.

anomaly associated with the eddies themselves rather than their environment, then both kinds of eddies correspond to length scales that are comparable to their deformation radii. Consequently, it is not entirely clear what are the dynamical differences between the two classes of eddies (if there are any) and, for simplicity, this study will refer to them as large ($\sim$50–100 km) and small ($\sim$10–20 km) eddies.

McWilliams (1985) has attributed the anticyclonic parity bias in the small eddies group to a formation process associated with localized diapycnal mixing. Other generation mechanisms such as those of D'Asaro (1988), Killworth and Davey (1989) and Nof (1990c) also favor the formation of anticyclones. In addition, high-amplitude anticyclones (i.e., lenses) are enclosed by a density surface, whereas high-amplitude cyclones are not. This suggests that the anomalies associated with the cyclones will decay much more rapidly than those associated with anticyclones because the diffusion does not have to overcome any density surfaces in the cyclonic case. Although the above processes are certainly plausible, the present study suggests a third possible parity bias mechanism. Even though this study has only focused on the conditions necessary (but perhaps not sufficient) for splitting to occur, it suggests that, *due to constraints imposed by angular momentum conservation, anticyclones are more likely to persist in the ocean because they cannot break up.*

Before completing the present discussion it is appropriate to mention that an opposite kind of parity bias has been observed in some laboratory experiments (Hopfinger et al. 1982; McEwan 1976), but this might be due to the relatively large Rossby number. Under such conditions, the anticyclones might have been highly (inertially) unstable and, hence, could not be formed in the first place. Inertial instability was also found to play a crucial role in the numerical stability of finite amplitude vortices where a cyclone–anticyclone asymmetry has been noted (McWilliams et al. 1986). An opposite kind of parity bias (i.e., more surviving cyclones) is also found in large eddies such as Gulf Stream rings. All of these points suggest that the parity bias is probably not universal but rather varies with the conditions in the field. This interesting aspect would be the subject of a different investigation.

7. Summary

This investigation considered: 1) the fission of single nonlinear baroclinic eddies (Figs. 1 and 2); 2) a combined fission–fusion process of (closely packed) multiple eddies consisting of two cyclones and two anticyclones (Fig. 6); and 3) the splitting of nonlinear joint eddies consisting of a lens with a cyclone on top (Fig. 11). The inviscid "reduced-gravity" model is nonlinear in the sense that both the Rossby number and the amplitude are of order unity. The transient splitting process is not directly addressed; instead, the initial and final states are connected (without solving for the breakup process) assuming that the final state is steady.

The connection between the two states is made using familiar principles (such as potential vorticity conservation) and the, somewhat less familiar, conservation of integrated angular momentum. As in Nof (1990a), the approximation of no angular momentum exchange with the environment [i.e., the fluid beyond the free bounding streamline (Figs. 1–3)] is made. However, here, the approximation is placed on a firmer ground by actually showing (using a not-so-trivial scaling analysis) that the exchange of angular momentum with the environment is small and negligible for *slow* splitting processes.

The results can be summarized as follows:

1) As in the barotropic case discussed by Nof (1990a), single (baroclinic) anticyclonic eddies cannot split even if energy is added to them. Baroclinic cyclones, on the other hand, can split if their intensity (potential vorticity) is above (below) a critical level (Fig. 4, upper panel). This results from the fact that baroclinic cyclones have more angular momentum than their anticyclone counterparts because they rotate in the same sense as the earth. Specifically, as in the barotropic case, it turns out that during the fission process the offspring acquire angular momentum so that only eddies with large angular momentum to begin with are capable of being potential parents.

2) When two nonlinear cyclones and two nonlinear anticyclones that are closely packed (Fig. 6) are allowed to interact, the cyclones break up and the anticyclones merge if the eddies' intensity falls within a "conversion" range (Figs. 7 and 9). Under such conditions the fission of the cyclones provides the energy necessary for the fusion of the anticyclones. This process and its final equilibrium corresponds to a situation which, in analogy to geostrophic turbulence, is referred to as ageostrophic turbulence.

3) Joint eddies consisting of a zero potential vorticity lens and a cyclone on top can also split and break up. Fission occurs whenever the cyclone's intensity (potential vorticity) is above (below) a critical level (Fig. 12, upper panel).

The main weakness of this analysis is that, although the detailed structure of the final state is solved for, the pattern and geometry of the final state are actually *assumed.* Hence, the validity of the result depends on the correctness of the assumed pattern. Nevertheless, the results are useful and may provide an additional explanation to the observed asymmetry in the percentage of anticyclones and cyclones. Specifically, it is suggested that the relatively high percentage of anticyclones in the ocean may be related to conservation of angular momentum and its crucial role in the splitting process.

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