Choked flows and wind-driven interbasin exchange

by Doron Nof

ABSTRACT

The classical question of how much water flows from one ocean to the other via connecting passages is addressed with a nonlinear analytical model. The focus is on gaps that are too broad to be influenced by the so-called "hydraulic control" and yet too narrow to allow free ("unchoked") flow through them.

We consider two rectangular oceanic basins, one containing a light upper layer overlying a slightly heavier deep layer and the other containing only one layer of fluid (whose density is identical to that of the lower fluid in the first basin). The basins are separated by a thin wall containing a gap which is initially blocked by a gate. All fluids are initially at rest and the pressure exerted on the gate corresponds to a sea-level difference (between the two basins) that is set up by the wind field. The conceptual gate is then removed and the resulting nonlinear flow from the inner basin to the outer basin is computed. The final steady state is taken to be analogous to the actual oceanic situation. The analytical calculations are based on an integrated momentum constraint which allows computation of the mass flow through the gap without solving for the rather complicated nonlinear flow within the gap itself and in its immediate vicinity.

It is found that the sea-level difference between the oceans drives a nonlinear flow (i.e., high amplitude and large Rossby number flow) parallel to the separating wall. Surprisingly, only about 40% of the generated upstream flow enters the gap. The remaining transport stays in the inner basin. A simple "gap formula" which enables one to compute the nonlinear transport via the gap is derived. In terms of the sea-level difference, the transport is \( \left(g'H^2/2f_0 \right)(1 - 1/e) ^2 \), where \( H \) is the undisturbed upper layer depth in the inner basin and the remaining notation is conventional. For the special case of no wind stress curl above the inner basin and no significant western boundary current, it is possible to relate the transport directly to the wind field. One finds that, for this particular case, the transport is independent of the stratification and is given by \( 0.3996 \int_0^L \tau_x^{(k)} \, dx / f_0 \rho \), where \( L \) is the width of the inner basin, \( \tau_x^{(k)} \) the zonal wind stress and \( \rho \) is the density of the water.

Qualitative "kitchen-type" laboratory experiments on a rotating table demonstrate that, as the theory predicts, only a fraction of the generated flow enters the gap. Quantitative numerical experiments using the Bleck and Boudra reduced gravity isopycnic model provide an even stronger support for the theory. They show that the analytically calculated transports are within 10–15% of the numerical calculations. Possible application of this theory to a number of passages such as the Windward Passage and the Indonesian throughflow is discussed.

1. Department of Oceanography 3048 and the Geophysical Fluid Dynamics Institute, The Florida State University, Tallahassee, Florida, 32306-3048, U.S.A.
1. Introduction

The exchange of water between oceans (or between oceans and marginal seas) is an important part of the general circulation and an interesting oceanographic problem. Historically, most of the theoretical examinations considered the connecting passages to have a channel-like structure with one-dimensional flows that are parallel to the channel walls (see e.g., Pratt, 1990 for a review). Many passages, however, have the geometry of an orifice or a gap rather than a channel. The importance of this aspect was recognized both by Garrett and Toulany (1982) who considered the time-dependent linear flow through gaps, and by Nof and Olson (1983) who considered the steady nonlinear transport.

Both of these studies demonstrated that, in the case of broad gaps, the concept of "hydraulic control" is not relevant and that, instead, the flow is "geostrophically controlled." This means that the maximum transport is given by the sea-level difference between the basins rather than by the internal Froude number reaching a value of unity in some connecting channel. The present article is an extension of the Nof and Olson (1983, hereafter referred to as NO) study in the sense that (i) a severely limiting assumption (regarding the transmission of information) made in NO is relaxed and replaced by a more sensible assumption related to the width of the gap; (ii) laboratory and numerical experiments which test the validity of the new theory are presented; and (iii) the computed transport is related to the wind field which sets up the pressure difference across the gap in the first place. An important aspect of the new theory is that only a fraction of the geostrophically controlled transport can actually enter the gap. Specifically, the transport that can enter a broad gap is only 40% of that allowed by the geostrophic control principle.

Consider the following situation as an idealized formulation of the problem. Two infinitely large basins are separated by a thin wall which contains a gap whose width is several times the deformation radius (Figs. 1 and 2). The inner basin contains two layers [a shallow upper layer whose density is \( \rho \) and an infinitely deep lower layer whose density is \( \rho + \Delta \rho \) where \( \Delta \rho / \rho \ll 1 \)] whereas the outer basin initially contains only one layer. The gap connecting the two basins is initially blocked by a gate which extends from the free surface to the interface. The lower layer in both basins is assumed to be in equilibrium. That is, the sea-level difference between the two basins is compensated for by the upper layer depth so that there are no pressure gradients in the lower layer and, consequently, there is no flow in the lower layer even though it is not blocked.

It is assumed here that the wind field is only important in terms of setting up the pressure difference between the basins. Namely, the wind is important when it operates on the basin scale \( L \) but unimportant for the much smaller gap scale (several times the Rossby radius). Similarly, the variation of the Coriolis parameter with latitude \( \beta \) is important when the general circulation in the basin is considered but unimportant when the gap dynamics are considered. For clarity, the connection
Figure 1. A schematic three-dimensional view of the conceptual model for the interbasin exchange. In this simple geometry, both the inner and the outer basins extend to infinity in the x and y directions. The sea level in the inner basin is higher than that of the outer basin by $H\Delta \rho /\rho$; this difference is later related to the wind field. An imaginary gate initially blocks the gap (whose width is much greater than the Rossby radius) which connects the two basins. The steady adjusted state reached after the removal of the gate is taken to be analogous to the oceanic situation.

between the wind field and the sea-level difference will not be dealt with until the gap dynamics have been described in detail.

At, say, $t = 0$ the gate is removed and, subsequently, some of the light water flows from the inner to the outer basin. Our aim is to determine the transport through the gap and to examine the currents that will be set up in the inner basin. It is expected that, after an initial period of adjustment, a steady state will be reached and it is this state that we will be looking for. To do so, we shall follow the NO (and Nof, 1993) approach in the sense that the integrated momentum constraint will be used to connect the upstream and downstream regions without solving for the detailed flow structure within the gap (Sections 2, 3, 4 and 5).

However, in contrast to NO, the limiting assumption of information transfer via Kelvin waves alone is relaxed here. Namely, in contrast to the NO conjecture that information can be transferred only via Kelvin waves propagating along an infinitely long wall, it is assumed here that (i) information can also be transmitted by Rossby waves and eddies interacting with the walls and (ii) the wall contains more than one gap so that Kelvin waves can also propagate clockwise around the wall separating the gaps. This implies that, unlike the NO case where region 2 (Fig. 3) could not be altered during the adjustment (because Kelvin waves could not reach it) all regions in the inner basin can now be reached. The above relaxation resolves the difficulty
Figure 2. Schematic two-dimensional diagram of the model initial conditions. $H$ is the undisturbed inner basin depth. The gap is initially blocked by a gate so that there is no flow from the inner to the outer basin.

associated with the breakdown of the NO solution for the configuration shown in Figures 1 and 2 where there is no initial current in the inner basin. It also introduces a whole new class of solutions where both regions 1 and 2 are set in motion; this new class of solutions is the focus of our present study.

A simple qualitative laboratory experiment on a rotating table was performed in order to examine the processes suggested by the mathematical solutions (Section 6). Specifically, an initially resting inner basin with two layers (i.e., a thin fresh water layer overlying a deep salty layer) and an outer basin with one salty layer were initially separated by a gate. After the removal of the gate the fresh water penetrated to the outer basin; the events associated with this penetration were photographed from above. As expected from the theoretical considerations, only a fraction of the current generated by the pressure difference across the gap enters the outer basin. Most of the generated flow remains in the inner basin and circulates in a clockwise
manner. Quantitative numerical experiments using the Bleck and Boudra reduced-gravity isopycnic model (with an infinitely deep lower layer) were also performed in order to assess the weaknesses of the theory (Section 7). As in the laboratory experiments, two basins separated by a wall containing a gap were used. We shall see that excellent agreement between the theory and experiments is found.

Our nonlinear solution is then related to the wind field which sets up the sea-level difference between the basins in the first place (Section 8). Finally, a qualitative application of the theory and experiment to the Windward Passage and the Indonesian throughflow is discussed in some detail.

Note that, as is frequently the case with such studies, there is some limited (unavoidable) overlap between the present article and NO (and Nof, 1993) because an attempt has been made to make the present paper self-contained. The reader who is only interested in the results may go directly to the end of Section 5. Readers who are familiar with the details of the NO analysis are warned in advance that, although

---

Figure 3. Schematic diagram of the final adjusted state. Since the transport of light water through the gap is finite, the current cross section in region 3 must also be finite so that the interface must intersect the free surface.
some of the following sections may appear to be similar to those in NO, there are some subtle differences between the two studies. These differences are related to the relaxation of the Kelvin waves assumption (mentioned earlier) and to a (compensating) front progression relationship.

2. Formulation

Consider again the configuration shown in Figures 1, 2 and 3. The wall separating the basins is taken to be thin so that, in the gap, no fluid can accumulate against it. The origin of our coordinate system is located at the edge of the gap. The \(x\) and \(y\) axes are oriented across and along the gap and the system rotates (uniformly) at \(f_0/2\) about the \(z\) axis. (The variation of the Coriolis parameter with latitude, \(\beta\), will be included later.) Initially, the gate, extending from the free surface to the interface, separates the two-layered inner basin from the one-layered outer basin. The level of the free surface in the inner basin at \(x \to \infty\) is higher than that of the outer basin by \(H \Delta \rho / \rho\) so that there is no flow in the lower layer even though the lower portion of the gap is not blocked.

At \(t = 0\) the gate is lifted and, subsequently, light fluid starts penetrating into the outer basin. After some time of \(O(f_0^{-1})\) a steady state will be reached and it is this state that we shall focus on. Away from the gap, the steady upstream and downstream currents are expected to be in geostrophic balance because, in these regions, the flow is one dimensional. Since the cross-sectional area of the flow which entered the gap must be finite, the penetrating flow must be adjacent to the right wall (looking downstream); there can be no such flow in region 4 which is adjacent to the left wall (see Fig. 3).

In contrast to NO where it was assumed that information can only be transferred by Kelvin waves (so that the field in region 2 remains unaltered), it is assumed here that Rossby waves and eddies interacting with the walls [i.e., anticyclones and cyclones translating to the left and right (looking off-shore)] can also transmit information (Shi and Nof, 1993; 1994). Such features are always present in the actual ocean and it is, therefore, appropriate to consider them even though they are not explicitly included in our calculations. This means that region 2 can also be altered so that our flow field consists of three unknown boundary currents (regions 1, 2 and 3) and a nonlinear region in the vicinity of the gap.

3. Governing equations and boundary conditions

All regions away from the gap (1, 2 and 3) are governed by the potential vorticity equation and the geostrophic relationship,

\[
\frac{\partial v}{\partial x} + f_0 = f_0 \frac{h}{H}
\]  

(1)
where $H$ is the initial upper layer undisturbed depth and the remaining notation is conventional. Note that the rigid lid approximation (which implies that the free surface vertical displacement is much smaller than the interface displacement) has been invoked.

The boundary conditions for regions 1 and 2 are,
\[ v \rightarrow 0 ; \hspace{0.5cm} x \rightarrow \infty \]  
\[ h \rightarrow H ; \hspace{0.5cm} x \rightarrow \infty \] (3) (4)
and for region 3,
\[ h = 0 ; \hspace{0.5cm} x = -\gamma_3. \] (5)

These conditions state that the velocities decay away from the walls and that the width of the flow in region 3 is not known in advance but rather must be found as a part of the solution.

In addition to these conditions, the Bernoulli function must be satisfied along the two streamlines that bound region 3 from the right and left (looking downstream) so that,
\[ (v^2/2 + g'h_3)_{x=0} = (v^2/2 + g'h_2)_{x=0} \] (6)
and,
\[ (v^2/2)_{x=-\gamma_3} - (v^2/2 + g'h_1)_{x=0}, \] (7)

where the subscripts 1, 2 and 3 indicate that the variable in question is associated with regions 1, 2 and 3. It can be easily demonstrated that the above boundary conditions are not sufficient to connect regions 1, 2 and 3 and close the problem. Closure can be achieved, however, by considering the constraints discussed in the following section.

4. Constraints

a. Mass. This familiar constraint can be written as,
\[ \oint vh \, dx = 0, \] (8)

where the arrowed circle indicates counterclockwise integration along the boundary shown in Figure 4. It will become clear later that this equation is automatically satisfied by the variables that satisfy (1–7). This results from the fact that we have applied the Bernoulli principle along the two bounding streamlines of the outer
Figure 4. A diagram of the integration area for the momentum equation. In the outer basin, the integration area is bounded by the wall, the free streamline ($\psi = 0, h = 0$) and a section across region 3. In the inner basin, the integration area extends well beyond the expected decay region (i.e., DE is located several deformation radii away from the walls). It is bounded by sections across region 2 and 1, the walls and the line DE which is parallel to the walls.

current so that, in fact, we have already stated that the two edges and walls are streamlines.

b. Integrated momentum. Although some readers may be familiar with this constraint, it is useful to derive it from first principles. Consider the region $S$ bounded by the dashed line shown in Figure 4. Multiplication of the $y$ momentum equation by $h$ and integration over $S$ gives,

$$\int \int_S \left[ hu \frac{\partial v}{\partial x} + hv \frac{\partial v}{\partial y} \right] dxdy + \int \int_S f_0 uh dxdy + \frac{g'}{2} \int \int_S \frac{\partial}{\partial y} (h^2) dxdy = 0,$$

which, by using the continuity equation and streamfunction $\psi$ can be reduced to,

$$\int \int_S \left[ \frac{\partial}{\partial x} (huv) + \frac{\partial}{\partial y} (hv^2) \right] dxdy - \int \int_S f_0 \frac{\partial \psi}{\partial y} dxdy + \frac{g'}{2} \int \int_S \frac{\partial (h^2)}{\partial y} dxdy = 0. \tag{9}$$
Application of Stokes’ theorem to (9) gives,
\[ \oint_{\phi} \rho u v \, dy - \oint_{\phi} \left( h v^2 + \frac{g' h^2}{2} - f_0 \psi \right) \, dx = 0, \]  
(10)
where \( \phi \) is the boundary of \( S \). Since at least one of the three variables \( u, v \) and \( h \) vanishes on every portion of the boundary \( \phi \), (10) reduces to
\[ \oint_{\phi} \left( h v^2 + \frac{g' h^2}{2} - f_0 \psi \right) \, dx = 0. \]  
(11)

Leaving the integrated momentum equation (11) aside for a moment, we note that, since the flow is geostrophic (in \( x \)) in both sections 1 and 2, it follows that,
\[ -f_0 \psi_1 + g' h_1^2 / 2 = c_1 \]  
(12a)
\[ -f_0 \psi_2 + g' h_2^2 / 2 = c_2, \]  
(12b)
where integration in \( x \) has been performed so that \( c_1 \) and \( c_2 \) are the, yet unknown, integration constants. We further note that at infinity (i.e., \( x \to \infty \)) both \( h_1 \) and \( h_2 \) go to \( H \) and \( \psi_1 \to \psi_2 \) implying that \( c_1 = c_2 \). Similar integration can be done for region 3. For simplicity, we define \( \psi \) to be zero along the front where \( h = 0 \) so that for region 3,
\[ -f_0 \psi_3 + g' h_3^2 / 2 = 0. \]  
(12c)

With the aid of (12) we now return to the integrated momentum equation (11). We recall that the flow is geostrophic in all the sections away from the gap and that \( \psi \) was defined to be zero along \( h = 0 \). These considerations lead to our desired integrated momentum constraint,
\[ \int_0^\infty h_1 v_1^2 \, dx + \int_0^\infty h_2 v_2^2 \, dx + \int_{-\gamma_3}^\gamma h_3 v_3^2 \, dx = 0. \]  
(13)

It is important to note that, even though the (uniform) Coriolis parameter \( f_0 \) does not explicitly appear in this integrated momentum constraint, there is an important fundamental difference between the rotating and the nonrotating constraint. The difference is that in the nonrotating case [i.e., relation (11) with \( f_0 = 0 \)] the pressure term \( g' h^2 / 2 \) does not drop out of the equation so that instead of (13) one obtains the familiar relationship,
\[ \int_0^\infty \left( h_1 v_1^2 + \frac{g' h_1^2}{2} \right) \, dx + \int_0^\infty \left( h_2 v_2^2 + \frac{g' h_2^2}{2} \right) \, dx + \int_{-\gamma_3}^\gamma \left( h_3 v_3^2 + \frac{g' h_3^2}{2} \right) \, dx = 0 ; \quad f_0 = 0. \]
Figure 5. The behavior of fluid in the central part of the gap during the adjustment. As a result of the gate's removal, fluid at \( x = 0 \) advances a distance \( \delta \) toward the outer basin.

Note that (13) cannot be reduced to the nonrotating case (upon substitution of \( f_0 = 0 \)) because the steps leading to it (12a–c) are invalid for \( f_0 = 0 \).

c. Conservation of momentum during the adjustment process. In this subsection we derive what will be later referred to as the “front progression constraint.” It provides the necessary closure condition and compensates for the relaxation of the NO Kelvin waves assumption.

Since the gap is much broader than the deformation radius, \( R_d \equiv (g' H_i)^{1/2}/f_0 \), fluid near the center of the gap does not feel the presence of the walls. Consequently, the adjustment of this fluid can be viewed as that of a stagnant fluid initially blocked by a very long gate (Fig. 5). In its final adjusted state, this fluid obeys, of course, the
potential vorticity equation and the geostrophic relationship (1) and (2). However, during the time-dependent adjustment it also obeys,

$$\frac{Dv}{Dt} + f_0u = 0$$  \hspace{1cm} (14)

because there are no variations in y. This equation is subject to the boundary condition \(v \to 0; h \to H\) as \(x \to \infty\). Since \(u\) can also be written as \(u = dx/dt\), it follows from (14) that,

$$\frac{D}{Dt} (v + f_0x) \equiv 0,$$  \hspace{1cm} (15)

which states that the front \((h = 0)\) must advance or retreat in order to generate long-front velocities \((v)\).

The solution of (1) and (2) is now,

$$v = (g'H)^{1/2} e^{-(x - \delta)/R_d},$$  \hspace{1cm} (16)

where \(\delta\), the distance that the front has advanced during the adjustment is given by,

$$\delta = R_d \equiv (g'H)^{1/2}/f_0.$$  \hspace{1cm} (17)

It was obtained by using the front progression condition (15) which yields,

$$(g'H)^{1/2} - f_0\delta = 0.$$  \hspace{1cm} (17a)

In deriving (14–17) it was assumed that the effects of \(\beta\) are small and can be ignored (i.e., \(\beta R_d/f_0 \ll 1\)).

We now return to the general problem where the walls are present and assume that, in the final adjusted state, there are no variations in the front position for all \(y \gg R_d\) (i.e., the width of the penetrated flow does not change for \(y \gg R_d\)). This immediately implies that

$$\gamma_3 = \delta = (g'H)^{1/2}/f_0,$$  \hspace{1cm} (18)

which, as we shall see, enables us to close the problem. Note that, as mentioned earlier, the main difference between the NO study and the present analysis is that (18) was not employed in NO because the assumption of a very broad gap (i.e., a gap that is several or many deformation radii and yet smaller than the basin scale) was not made there. (NO only required that the gap be of the order of the Rossby radius.) In most cases, the deformation radius is 20–40 km (and the basin scale is 1000–10,000 km) so that gaps that are 100–300 km broad satisfy our present choice of scales.
5. Solution

a. General solution for regions 1, 2 and 3. Since (1) and (2) can be combined to

\[ \frac{\partial v^2}{\partial x^2} - \nu/R_d^2 = 0, \]

the general solution for regions 1 and 2 (obeying the condition \( v \to 0 \) as \( x \to \infty \)) is,

\[ v_1 = A_1 e^{-x/R_d}; \quad h_1 = H \left( 1 - \frac{A_1}{f_0 R_d} e^{-x/R_d} \right), \tag{19} \]

\[ v_2 = A_2 e^{-x/R_d}; \quad h_2 = H \left( 1 - \frac{A_2}{f_0 R_d} e^{-x/R_d} \right), \tag{20} \]

where \( A_1 \) and \( A_2 \) are constants to be determined from the boundary conditions.

Similarly, the general solution for region 3 is,

\[ v_3 = A_3 e^{-x/R_d} + B_3 e^{x/R_d} \tag{21a} \]

\[ h_3 = H \left( 1 - \frac{A_3}{f_0 R_d} e^{-x/R_d} + \frac{B_3}{f_0 R_d} e^{x/R_d} \right). \tag{21b} \]

The general solutions (19–21) contain four unknowns \( A_1, A_2, A_3 \) and \( B_3 \) so that four equations are needed to solve the problem. There are three boundary conditions that we have not yet used (5, 6 and 7) and the integrated momentum constraint (13) provides the fourth necessary equation.

b. Detailed solution. To obtain the four unknowns \( A_1, A_2, A_3 \) and \( B_3 \), relations (18–21) are substituted into (5), (6), (7) and (13). One finds,

\[ 1 - \hat{A}_3 e + \hat{B}_3 e^{-1} = 0 \tag{22} \]

\[ (\hat{A}_3 e + \hat{B}_3 e^{-1})^2 = \hat{A}_1^2 - 2\hat{A}_1 + 2 \tag{23} \]

\[ (\hat{A}_3 \hat{B}_3)^2 = \hat{A}_2^2 + 2(\hat{A}_3 \hat{B}_3) \tag{24} \]

\[ \frac{1}{2} (\hat{A}_1^2 - \hat{A}_3^2) + \frac{1}{3} (\hat{A}_3^3 - \hat{A}_1^3) - \hat{A}_3^2 (e^2 - 1)/2 + \hat{B}_3^2 (e^{-2} - 1)/2 - 2\hat{A}_3 \hat{B}_3 \]

\[ + \hat{A}_3^3 (e^3 - 1)/3 + \hat{A}_3^3 \hat{B}_3 (e - 1) + \hat{B}_3^3 (e^{-3} - 1)/3 + \hat{A}_3 \hat{B}_3^2 (e^{-1} - 1) = 0, \tag{25} \]

where all the unknowns have been nondimensionalized by \((g'H)^{1/2}\) and denoted by a hat \( (\hat{\cdot}) \). Although these equations are not simple, inspection shows that the exact solution is,

\[ \hat{A}_1 = 1; \quad \hat{A}_3 = \hat{A}_2; \quad \hat{B}_3 = 0; \quad \hat{A}_2 = e^{-1}, \tag{26} \]

and that there are no other physically relevant solutions.
In dimensional form the complete solution is,

\[
\begin{align*}
v_1 &= (g'H)^{1/2} e^{-x/R_d}; \quad x \geq 0 \\
h_1 &= H(1 - e^{-x/R_d}); \quad x \geq 0 \\
v_2 &= (g'H)^{1/2} e^{-x/(R_d-1)}; \quad x \geq 0 \\
h_2 &= H(1 - e^{-x/(R_d-1)}); \quad x \geq 0 \\
\gamma_3 &= R_d \\
v_3 &= (g'H)^{1/2} e^{-x/R_d}; \quad x \leq 0 \\
h_3 &= H(1 - e^{-x/R_d}); \quad x \leq 0
\end{align*}
\]

and the corresponding transports are,

\[
\begin{align*}
T_1 &= \frac{g'H^2}{2f_0}; \\
T_2 &= \frac{g'H^2}{2f_0} \left[ 1 - \left( \frac{1 - 1}{e} \right)^2 \right] = 0.6004T_1 \\
T_3 &= \frac{g'H^2}{2f_0} \left( 1 - \frac{1}{e} \right)^2 = 0.3996T_1
\end{align*}
\]

This implies that only about 40% of the total induced transport goes through the gap. The remaining 60% circulates in the inner basin and never enters the gap. It is worth pointing out here that, for the case under study where there is no initial current in the inner basin, the NO solution breaks down.

6. Laboratory experiments

To examine the validity of the foregoing theory a set of qualitative "kitchen-type" laboratory experiments were performed (Fig. 6).

a. Experimental procedures. The apparatus was centered to less than ±1 mm of the table rotation axis. A 35 mm motor-driven camera was mounted on the rotating table and the exchange process was photographed (from above) every few seconds. The stratified tank was rotated counterclockwise (with the gate closed) at a uniform rotation rate until the system reached a solid body rotation (about 30 min). The experiment began with the removal of the gate. After a few seconds the currents system away from the gap reached an approximately geostrophic balance. Note that, since the laboratory basin is, obviously, finite, region 2 could be reached by Kelvin waves propagating counterclockwise along the periphery. The influence of laboratory friction, though clearly greater than the actual friction in the ocean, was not significant. The exchange process was observed for about 20 sec, whereas the
Figure 6. A sketch of the experimental apparatus. The interbasin exchange is set up by removing the gate which initially separates the two-layered ocean (on the right) from the one-layered ocean (on the left). The “wiggly” arrow indicates propagation of the current.

exponential spindown time scale (i.e., the decay to $1/e$ of the initial light fluid depth) was estimated to be much longer—about 10 min.

b. Interaction with the lower layer. A potentially serious difference between our laboratory experiments and the computations presented earlier stems from the fact that the calculations were made for an infinitely deep environmental fluid, whereas the experiment is associated with a finite environmental fluid. Specifically, the currents resulting from the removal of the gate will always induce some motion underneath. Because of the depth ratio in our experiment (about $1/6$), the role of the induced motion underneath is probably not entirely negligible. It is not expected, however, to be crucial to the processes in question.
c. Results. Subsequent photographs of a typical experiment are shown in Figures 7–8. The gate is removed (slightly before the state marked as $t = 0$) and the light dyed fluid immediately starts penetrating into the outer basin. Even though the inner basin was initially at rest, a current is established in region 2 within a few revolutions. This can be easily verified by tracing the clusters of aluminum particles marked on the photographs. It is in agreement with our predictions which, for this particular case, rely on the fact that in an enclosed basin it is not necessary for either Rossby waves or eddies to be present because Kelvin waves can reach region 2 by propagating along the boundary in a counterclockwise manner. The drawback of this situation is that by the time the (internal) Kelvin waves reach region 2, the inner basin has already lost a large fraction of the fluid that it can lose so that a truly steady state cannot be reached.

It is interesting to note that our assumption of one-dimensionality within the gap is violated after some time because of a vortex generated near the downstream edge. The vortex is probably a result of horizontal friction (next to the walls) which is absent from our theory. This is supported by our numerical experiments (Section 7) which do not have any friction next to the wall and, consequently, do not produce such an eddy (see Fig. 11). A thorough examination of this question is beyond the scope of the present study.

7. Numerical experiments

To further examine the validity and weaknesses of our analytical solution, quantitative numerical experiments using the Bleck and Boudra reduced-gravity isopycnic model [with an infinitely deep lower layer (see e.g., Chassignet and Cushman-Roisin, 1991; Bleck and Boudra, 1986)] were performed. To do this, we used a closed inner basin of approximately 1200 × 3500 km and a closed outer basin of approximately 3500 × 3500 km (Fig. 9). The upper layer undisturbed depth in the inner basin was approximately 150 m, the “reduced gravity” was $10^{-2}$ m sec$^{-2}$, and the Coriolis parameter was $2.5 \times 10^{-5}$ sec$^{-1}$. These give a Rossby radius of about 50 km and we used a gap that is 250 km broad. The horizontal eddy viscosity was $2 \times 10^2$ m$^2$ sec$^{-1}$, the grid spacing was 6 km (in both the $x$ and the $y$ direction) and the time step was 360 sec. The boundaries were slippery and, as is frequently done, the vorticity was taken to be zero next to the walls.

The continuity equation was solved using the Flux Corrected Transport. To allow the information to quickly reach region 2, an additional secondary gap was placed several hundred kilometers downstream of the main gap (Fig. 9). [By “downstream” we mean here to the left of the main gap (looking toward the inner basin).] Such a secondary gap allows Kelvin waves to propagate clockwise around the wall separating the two gaps and reach region 2 within 10–20 rotation periods after the removal of the gate. To delay the return of the already withdrawn fluid to the main gap, a spiraling wall (a few thousand kilometers long) was added to the outer basin. Such a
Figure 7. Subsequent photographs of a typical experiment. The dyed light fluid starts penetrating into the outer basin when the gate is lifted. The white ring is an (unavoidable) reflection of the fluorescent light shining from above. At $t = 0$ the white ring is still distorted due to gravity waves generated by the removal of the gate. Such waves disperse and change into Kelvin waves within a few seconds. It is clear that, as the theory predicts, a counterclockwise flow pattern is established in the inner basin even though the basin was initially at rest. This can be easily seen by following the clusters of aluminum particles surrounded by the marked (small and large) circles, and the marked open square. Physical constants: $f = 1.26 \text{ sec}^{-1}; \Delta \rho/\rho = 0.01$. 

[f] $t = 0$ 
[b] $t = 2.4 \text{ sec}$ 
[c] $t = 7.2 \text{ sec}$ 
[d] $t = 9.0 \text{ sec}$
Figure 8. The same as Figure 7 but for later times.
Figure 9. Schematic diagram of the configuration used for the numerical experiment. The secondary gap allows Kelvin waves to propagate clockwise around BC and reach region 2 within several rotation periods. The spiral traps the already withdrawn fluid (which propagates counterclockwise around the outer basin) and, hence, delays its return to the inner basin.

The spiral extends the withdrawal time to several hundred days allowing us to complete the experiment before the fluid returns to the inner basin.

It is important to note that the adjustment process involves a considerable amount of energy loss. In the analytical model, part of the energy is removed by the radiation of waves but energy is not removed in the numerical model. This implies that oscillations are always present in the runs. Figure 10 shows that, despite these

Figure 10. A comparison between the analytical (straight thick lines) and numerical (thin wiggly curves) solution for the gap problem. The (nondimensional) numerically calculated transports in the various regions are shown as a function of time. The model used is the 1½ layer Bleck and Boudra isopycnic model (see text for details). The initial delay associated with the establishment of the currents reflects the time that it took for the information to reach region 2. Note that, given the simplicity of the analytical model and the lack of a mechanism to remove excess energy from the numerical model, the agreement is excellent.
Figure 11. Depths contours in the vicinity of the main gap 180 days (i.e., 290 rotation periods) after the removal of the gate. The deformation radius is approximately 50 km. Note that, as assumed in the analytical model, the flow is indeed one dimensional in all regions away from the gap.

oscillations and despite the fact that there is no friction in the analytical model, the average transports agree remarkably well with the nonlinear theory. Figure 11 illustrates that, as assumed in the analytical model, away from the gap the flow is indeed one dimensional.
8. Applications and discussion

The foregoing theory might be applicable to numerous passages because many gaps which separate adjacent oceanic basins are broad and short. Two passages for which there is widely published data are the Windward Passage connecting the Atlantic Ocean and the Caribbean Sea and the Indonesian passages which connect the Pacific and the Indian Ocean. We shall focus on these two passages.

a. Relation to the wind field. To make our application complete, it is necessary to relate the pressure difference between the basins (which enter our problem through the undisturbed depth \( H \)) to the wind field. To do so, we position our model so that the gap and the separating walls are meridional, and consider the following familiar model for the general oceanic circulation,

\[
-fV = \frac{g'}{2} \frac{\partial}{\partial x} (h^2) + \frac{\tau_{s}^{(v)}}{\rho} \tag{29}
\]

\[
fU = -\frac{g'}{2} \frac{\partial}{\partial y} (h^2) KV, \tag{30}
\]

where \( f = f_0 + \beta y \), \( U \) and \( V \) are the vertically integrated transports in the \( x \) (east) and \( y \) (north) directions, \( \tau_{s}^{(v)} \) is the surface wind stress in the \( x \) direction, \( \rho \) the water density, and \( K \) is the coefficient of interfacial friction [i.e., \( K = \alpha \beta w \), where \( w \sim O(R_d) \) is the width of the boundary current and \( \alpha \) is a coefficient of order unity]. Eq. 29 holds both in the sluggish ocean interior away from the western boundary and in the western boundary current where the flow is geostrophic in the cross-stream direction. In this model, energy is supplied by the wind over the entire ocean and is dissipated through interfacial friction within the western boundary current. Since the associated frictional force is directed along the boundary it is not present in (29).

Integration of (29) from the western to the eastern boundary gives the (square of the) sea-level difference,

\[
\frac{g'}{2} (h_w^2 - h_e^2) = \frac{1}{\rho} \int_0^L \tau_{s}^{(v)} \, dx,
\]

where the subscripts “\( w \)” and “\( e \)” denote association with the western and eastern boundaries. \( L \) is the basin’s length (\( L \gg R_d \)) and it has been assumed that there is no net transport within the cross-section (i.e., the boundary current transport cancels the Sverdrup transport). Elimination of the pressure term between (29) and (30) gives,

\[
\beta V = -\frac{1}{\rho} \frac{\partial \tau_{s}^{(v)}}{\partial y} - K \frac{\partial V}{\partial x},
\]
Figure 12. Side view of the wind set-up model. Here, the basins are of equal length and the upper layer depth vanishes along the eastern boundaries.

which also can be integrated across the basin to give

\[ V_w = -\frac{1}{K \rho} \int_0^L \frac{\partial \tau_y^{(x)}}{\partial y} \, dx, \]  

(32)

where the transport along the eastern boundary has been neglected and \( V_w \) is the vertically integrated transport (of the boundary current) along the western wall.

The simplest possible case that we can consider is that where there is no significant wind stress curl (\( \partial \tau_y^{(x)}/\partial y = 0 \)) so that there is no significant western boundary current (\( V_w = 0 \)). Under such conditions, our approximation of no initial current in the inner basin (made earlier in our development) is appropriate. Our separate assumption of no initial upper layer in the outer basin implies that we need to consider the special case where the thermocline vanishes along the eastern boundary. This is probably not such a bad approximation because it is the difference in the square of the depths (along the separating wall) that is taken into account in the transport calculation. For basins with equal length this difference is identical to that given by \( h_e = 0 \) (see Fig. 12).

Under such conditions, a combination of (31) and (28) gives the simple useful relationships for the transports.

\[ T_1 = \int_0^L \tau_y^{(x)} \, dx/f_0 \rho \quad ; \quad T_2 = 0.6004 \int_0^L \tau_y^{(x)} \, dx/f_0 \rho \]

\[ T_3 = 0.3996 \int_0^L \tau_y^{(x)} \, dx/f_0 \rho. \]  

(33)
Recall that $T_1$ is the (inner basin) northward transport upstream, $T_3$ the transport through the gap and $T_2$ is the remaining inner basin transport downstream (Fig. 3). As mentioned, even though there was no boundary current in the inner basin to begin with and there is no wind stress curl, a nonlinear boundary current is established due to the gap.

This completes our derivation for the transports as a function of the wind stress. We shall now proceed and discuss its application.

\[ b. \textit{The Windward Passage.} \] This passage is located approximately at 18N (corresponding to $f_0 = 1.3 \times 10^{-5} \text{ sec}^{-1}$) and the annual average wind stress across the Atlantic Ocean to the east is about 1.5 dyn/cm² (see e.g., Hellerman and Rosenstein, 1983). Taking into account an oceanic width of $L = 6000 \text{ km}$ and the above parameters, relation (33) gives, $T_1 = 20 \text{ Sverdrup}$, $T_2 = 12 \text{ Sv}$, and $T_3 = 8 \text{ Sv}$. These values are in qualitative agreement with previous estimates (see e.g., NO). Also, the directions of all the currents (upstream and downstream) agree with the observed flows, i.e., the origin of the transport is in the northeastern part of the inner basin (region 3, north of Hispaniola) and the remaining transport flows to the northwest (region 2, north of Cuba).

Note that the advantage of our newly suggested method of computation is that, in contrast to most methods (e.g., NO), it does not require a detailed knowledge of the thermocline structure in the vicinity of the gap (i.e., depth and stratification), nor does it require an estimate of the flow in region 2 (needed in the NO method).

\[ c. \textit{The Indonesian throughflow.} \] The entrance to the Indonesian passages is approximately at 5N (corresponding to $f_0 = 1.3 \times 10^{-5} \text{ sec}^{-1}$) and the annual average wind stress across the Pacific Ocean to the east is about 0.4 dyn/cm². Taking into account a width of $L = 16,000 \text{ km}$, (33) now gives, $T_1 = 50 \text{ Sv}$, $T_2 = 30 \text{ Sv}$, and $T_3 = 20 \text{ Sv}$. As in the Windward Passage case, the predicted amount of water flowing through the passages ($T_3$) agrees roughly with the observed values (see e.g., Wyrtki, 1961; Piola and Gordon, 1984; Fine, 1985; Fu, 1986; Meyers et al., 1995, and Fieux, 1994). However, in contrast to the Windward Passage case, the predicted origin of the throughflow does not agree with the observations. The above theory predicts that the throughflow originates in the South Pacific whereas observations suggest a North Pacific origin (Fine, 1985; Lukas et al., 1991; Ffield and Gordon, 1992).

Interestingly, the linear analytical computations of Godfrey (1989), Godfrey et al. (1993) and Wajisnowicz (1993a,b), all of which are based on the so-called “island rule,” also suggest a southern origin for the throughflow. They argue that the discrepancy between their linear theory and the observations is due to strong horizontal diffusion acting over long distances which masks the actual origin of the throughflow and gives the impression that the source is the North Pacific. While this is certainly possible, it appears that, in our presently proposed nonlinear model, it is
the neglect of the pre-existing (southward flowing) western boundary current in the Pacific that is the culprit. Such a current can be incorporated by including the curl of the wind stress but this is not as straightforward as it may seem to be because the current enters both the sea-level difference and the front progression condition. Attempts to include these processes in our computation are presently being made and will be reported elsewhere.

d. Conclusions. The primary aims of both our theory and experiment were (i) to establish a way of computing the transport through broad gaps in terms of the wind-stress over the adjacent ocean, and (ii) to determine the associated currents in the vicinity of the gap. The inviscid exchange process is viewed as being the result of opening a gate separating two initially unbalanced basins (Figs. 1, 2). Attention is focused on the final steady state resulting from the gate removal (Fig. 3); the process is highly nonlinear because both the amplitude and the Rossby number are of order unity. The key to the construction of the nonlinear solution is the employment of the integrated momentum constraint (eq. 13) which enables one to connect the upstream and downstream regions without solving for the gap itself.

Our findings are:

1. An exact nonlinear analytical solution to the adjustment problem (Fig. 2) can be constructed even if the Nof and Olson (1983) assumption of no alteration to region 2 (Fig. 3) is relaxed. Although the Nof and Olson assumption that no Kelvin waves can reach region 2 is reasonable, Rossby waves and eddies are always present in the actual ocean. Such features can, of course, transfer information to region 2 and alter its structure. Also, most gaps have neighboring gaps so that they are not isolated. Such adjacent gaps allow the information to reach region 2 by Kelvin waves propagating anticyclonically around the island separating the gaps (e.g., Cuba).

To compensate for the relaxation of the NO condition of no alteration to region 2, it is assumed that (i) the gap is broad enough so that fluid in the center of the gap does not feel the presence of the walls, and that (ii) the width of the penetrated current does not vary as one proceeds from the downstream edge of the gap to region 3.

2. With the above approximations, the transport through the gap is 0.3996 \((g' H^2/2f_0)\) which is only 40% of the total upstream transport generated by the adjustment.

3. Laboratory experiments clearly support the concept that only a fraction of the generated upstream transport flows through the gap (Figs. 7, 8). In the laboratory, the transfer of information (to region 2) occurred via Kelvin waves traveling counterclockwise along the circumference. Such waves are obviously absent from many oceanic situations (because the basins are not closed) but, as
mentioned, in the ocean, eddies and Rossby waves play a role equivalent to that of the laboratory Kelvin waves and there is usually more than one gap in the boundary so that Kelvin waves can reach region 2 by propagating clockwise around the land that separates the gaps.

4. Quantitative numerical experiments using the Bleck and Boudra reduced-gravity isopycnic model provide a still stronger support for the theory. Such experiments were done by removing a conceptual partition separating two basins (Fig. 9) and studying the resulting time-dependent adjustment process. The numerically predicted transports are in excellent agreement with the analytical predictions (Fig. 10). The assumption of one dimensionality away from the gap is also confirmed (Fig. 11).

5. For the special case of no wind stress curl in the adjacent ocean (i.e., $\partial \tau_x(y)/\partial y \equiv 0$ implying no pre-existing western boundary current) it is possible to relate our transport computations directly to the wind field (Fig. 12). One finds that the nonlinear transport through the gap is simply $0.3996 \int_0^L \tau_x \, dx/f_0 \rho$ (where $\tau_x$ is the zonal wind stress and $\rho$ is the water density) and that $0.6004 \int_0^L \tau_x \, dx/f_0 \rho$ flows as a boundary current in the inner basin. This implies that gaps may be partially responsible for the generation of western boundary currents and that numerical models which ignore gaps may not provide us with the correct answers.

6. Application of the above theory to the Windward Passage shows that both the observed transport and the observed origin of the water (southeast of the gap) agree qualitatively with the theoretical predictions. This is probably due to the fact that, indeed, there is no pre-existing western boundary current northeast of Cuba and Hispaniola. Namely, most of the westward flow in the Atlantic and the resulting western boundary current enter the Caribbean much farther to the south; this flow later penetrates into the Gulf of Mexico and eventually forms the Florida Current and the Gulf Stream.

7. Application of the theory to the Indonesian throughflow, on the other hand, shows that while the observed transport agrees qualitatively with the theoretical prediction, the origin of the actual throughflow appears to contradict the theoretical prediction. Interestingly, this somewhat problematic aspect of our nonlinear theory agrees with some previous linear theoretical predictions. It is believed that the discrepancy between all of these theories and the observations results from the neglect of the nonlinear Mindanao Current. That is to say, the discrepancy is probably due to our neglect of the wind stress curl and the resulting pre-existing western boundary current which approaches the gap from the north and cannot be ignored. This latter aspect is presently being investigated and will be reported elsewhere.
Acknowledgments. Conversations with M. E. Stern and W. K. Dewar were very useful. The laboratory experiments were performed in the Geophysical Fluid Dynamics Institute at Florida State University with the help of Sergey Borisov; the apparatus was constructed by Jim Winne. The numerical integrations were conducted by Steve Van Gorder. This study was supported by the National Science Foundation (NSF) under contracts OCE 9012114 and OCE 9102025, and the Office of Naval Research (ONR) under contract N00014-89-J-1606. This is contribution number 361 of the Geophysical Fluid Dynamics Institute.

REFERENCES


Received: 15 February, 1994; revised: 27 May, 1994.