Why are rings regularly shed in the western equatorial Atlantic but not in the western Pacific?

DORON NOF

Department of Oceanography 3048 and the Geophysical Fluid Dynamics Institute, The Florida State University, Tallahassee, Florida 32306-3048, USA

Abstract — The western equatorial Atlantic is characterized by the formation and shedding of 3-4 large anticyclonic rings per year. These rings originate from the North Brazil Current which, in response to the vanishing wind stress curl (over the ocean interior), retroreflects and turns eastward at around 4°N. After their formation and shedding the rings propagate toward the northwest along the South American coast carrying an annual average of about 4 Sv. As such, the rings constitute an important part of the meridional heat flux in the Atlantic.

The same cannot be said, however, of the western equatorial Pacific. Here, the situation is entirely different even though the South Equatorial Current retroreflects at roughly the same latitude as its Atlantic counterpart, the North Brazil Current. Although the South Equatorial Current retroreflection is flanked by two quasi-permanent eddies (the so-called Halmahera and the Mindanao eddies), these eddies are an integral part of the current itself and are not shed. Consequently, they are not associated with any meridional heat flux. An important question is, then, why the two oceans behave in such a fundamentally different way even though the source of the rings, the retroflected currents, are very similar in the two oceans.

To answer this question, the two oceans are compared using recently developed analytical and numerical models for the western equatorial oceans. It is first pointed out that, according to recent developments in the modelling of the western equatorial Atlantic, the North Brazil Current retroreflection rings are formed, shed and drift to the west because, in the Atlantic, this is the only way by which the momentum flux of the approaching and retroreflecting current can be balanced. In this scenario, the northwestward flow force exerted by the approaching and retroreflecting North Brazil Current (analogous to the force created by a rocket) is balanced by the southwestward force exerted by the rings as they are formed (analogous in some sense to the kickback associated with a firing gun).

On the other hand, in the western equatorial Pacific, the formation and shedding of rings is unnecessary because the southward flowing Mindanao Current provides an alternative mechanism for balancing the northward momentum flux of the South Equatorial Current. This implies that it is the absence of a counter current (such as the Mindanao) in the western Atlantic that causes the formation and shedding of North Brazil Current rings. A remaining difficulty with the above scenario is that most colliding and retroreflecting currents (i.e. the Mindanao and South Equatorial currents) are not “balanced” in the sense that they cannot be stationary but rather must drift along the coast. It is shown that, in the case of the western Pacific, the long-shore migration is arrested by the Indonesian Throughflow which allows the “unbalanced” fraction of the approaching currents to leak out into the Indian Ocean. This resolves the above difficulty and allows the retroflection to be approximately steady. © 1997 Elsevier Science Ltd
1. INTRODUCTION

The equatorial Atlantic and the equatorial Pacific play an important role in the world climate. Cross-equatorial flows in these oceans are of particular interest because of the associated meridional heat flux which has a direct influence on the global heat budget. It is known that such flows are subject to a potential vorticity constraint resulting from the change in planetary vorticity which accompanies any meridional flow. Since flows in the ocean interior (i.e. away from western boundaries) are slow, their relative vorticity is small so that it cannot compensate for the change in the planetary vorticity required by equatorial crossing. Consequently, the fluid in the ocean interior cannot be easily displaced from one hemisphere to the other. Most cross-equatorial flows occur via western boundary currents where the relative vorticity is high so that it can compensate for the change of planetary vorticity. It is for this reason that we focus here on the upper waters of the western equatorial oceans.

Despite their geographical similarity, the Atlantic and the Pacific oceans display very different dynamics (Fig. 1) and the purpose of this review article is to compare the processes that control each ocean. The article threads together several recently developed theories. It points out that the two oceans are very similar in terms of the cross-equatorial currents which retroflect and turn

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1The reader who is interested in the origin of the term "retroflection" is referred to Lutjeharms, de Ruijter and Peterson (1992). It was first used by Bang (1970) who borrowed it from the medical field to describe a current that curves back on itself. In the medical field it is frequently used to describe the bending back of an organ (such as a uterus) upon itself.
eastward at around 4°N in both oceans. There are, however, significant differences in terms of eddy shedding from these two currents. These differences can be traced to the Mindanao Current and the Indonesian Throughflow which do not have a counterpart in the Atlantic. The article can be thought of as an assemblage of a number of individual puzzles, each of which addresses an individual process in one of the oceans (e.g. NOF and PICHEVIN, 1996; LEBEDEV and NOF, 1996, 1997; NOF, 1996) into one large puzzle that addresses both oceans together.

We shall first present the recent theory of NOF and PICHEVIN (1996, hereafter referred to as NP) which explains why North Brazil Current (NBC) rings are generated (Section 2). We shall show that these rings are not generated by the classical instability of a zonal current but rather by the curving and turning of the retroreflecting NBC. We shall further show that rings are generated because, in the western Atlantic, they are the only feature that can balance the momentum flux of the retroreflecting NBC.

We shall then proceed to the western Pacific and present the recent theory of NOF (1996) which shows that, here, the momentum flux of the retroreflecting South Equatorial Current (SEC) is balanced by a branch of the southward flowing Mindanao Current (MC). In this case some of the SEC water and some of the MC water leaks out into the Indian Ocean through the Indonesian Passages (Section 3). We shall then demonstrate that Indonesian Passages play a crucial role in the dynamics of the western equatorial Pacific because without the above leakage the SEC-MC confluence zone could not have been stationary (LEBEDEV and NOF, 1996, 1997, hereafter referred to as LN). Instead, it would have drifted along the coast as is the case with the confluence zone in the South Atlantic where there is a strong seasonal and interannual variability. Finally, the results are discussed and summarized in Section 4.

2. WHY ARE RINGS GENERATED BY THE RETROFLECTING NORTH BRAZIL CURRENT?

In this section we shall examine the classical question of what happens when a warm western boundary current, such as the North Brazil Current (NBC) retroreflects. The traditional view is that the northwestward flowing current separates from the wall, turns to the right (looking offshore), and forms a zonal boundary current that flows eastward. We shall see shortly, however, that integration of the steady inviscid momentum equation along the boundary gives the long-shore momentum flux (or flow-force) and shows that such a scenario leads to a paradox. To resolve the paradox the separated current must constantly shed anticyclones which propagate to the northwest because of β.

To show this, NP constructed a nonlinear analytical solution based on the idea that nonlinear periodic flows can be integrated over a control volume and a generation period. It turns out that the problem involves a new eddy retroreflection length scale \( R_d \epsilon^{1/6} \) (where \( R_d \) is the parent current Rossby radius and \( \epsilon = \beta R_d f_0 \) that is greater than that of most eddies \( R_d \)). Namely, while most eddies, such as Gulf Stream rings, have the length scale of their parent current (as a result of the meandering process which forms the rings), our eddies have a much larger scale. In our scenario the intense (inertial) parent current has a length scale of the same order as its own Rossby radius \( R_d \left( (g' H f)^{1/4} \right) \), where \( g' \) is the reduced gravity \( (gA \rho/\rho) \) and \( H \) is the upstream undisturbed depth (i.e. the depth away from the wall where the velocity vanishes), and the intense eddies have a length comparable to their own Rossby radius \( R_{de} \) (based on their depth scale \( H_e \)). Note that the association of the features length scale with their Rossby radius is a result of velocity scales which are \( -O(g'H) \) for the current and \( -O(g'H_e) \) for the eddies. Calculations show that, for the retroflected NBC which transports about 45 Sv,
Fig. 1a. Schematic diagram showing the western equatorial Atlantic and the formation of a North Brazil Current (NBC) retroflection eddy (based on images shown by JOHNS et al., 1990). Three to four times each year starting in July the NBC retroflection advances northwestward along the boundary to about 9°-10°N forming a current loop. The sides of the loop come together, and the current loop pinches off from the main current to form a discrete eddy. As the eddy separates, the retroflection forms further south near 5°-6°N. Retroflection eddies are about 400km in overall diameter and drift northwestward at about 10-12 cm/s. "Wiggly" arrows denote migration and are distinguished from solid arrows which denote flow direction. Adapted from RICHARDSON et al. (1994).

Fig. 1b. The flow pattern in the western equatorial Pacific (adapted from FFIELD and GORDON, 1992). The North Equatorial Current (NEC) and the resulting Mindanao Current (MC) approach the passages from the north, whereas the South Equatorial Current (SEC) approaches the passages from the south. The North Equatorial Counter Current (NECC) carries water to the east. The Halmahara eddy (HE) and the Mindanao eddy (ME) do not usually drift away from their generation area.
eddy's shed approximately once every 90 days. In what follows, NP’s results are briefly reviewed.

### 2.1 Observational background

As already mentioned, the suggestion that the western tropical Atlantic is important to the meridional heat flux comes from the observation that at subtropical latitudes western boundary currents represent an important component of the global heat budget (Bryden and Hall, 1980). Observations imply that, at the surface, the summer and fall circulation in the western tropical Atlantic is dominated by the retroflection of the NBC which carries up to 50 Sverdrups of surface water to the east (Brown, Johns, Leaman, McCready, Molinari, Richardson and Rooth, 1992). The water immediately below the thermocline also retroflects (Metcalf and Stalcup, 1967) but at a different latitude. During other periods of the year (i.e. winter and spring) the NBC does not retroflect.

The observations of Johns, Lee and Schott (1990), Didden and Schott (1993), Richardson, Hufford, Limeburner and Brown (1994) and Fratantoni, Johns and Townsend (1995) all suggest that the NBC retroflection sheds at least three eddies per year. Through the eddies’ migration, about 3-4 Sv (20% of the total upper-ocean cross-gyre transport required by the Atlantic thermohaline overturning - Rintoul, 1991) is annually transported to the north Atlantic. As is the case with most rings resulting from retroflecting currents, NBC rings are larger than rings such as Gulf Stream or Kuroshio rings. They have a diameter of roughly 400-600km and a depth of 400-800m; they migrate at a rate of 10-12 cm s⁻¹ along the coast (Fig. 1). Gulf Stream and Kuroshio rings, on the other hand, have a smaller diameter of 200-300km and migrate at a rate of a few centimeters per second. The large size of the NBC rings and the fact that these rings are always formed at the same location suggests that, in contrast to many rings, they are probably not formed by an instability process². Instead, their generation is probably a result of the particular structure of the current which loops and curves back on itself. For other issues related to these rings the reader is referred to Bruce and Kerling (1984), Bruce (1984), Bruce, Kerling and Beatty (1985) and Cochrane, Kelly and Olling (1979).

### 2.2 Theoretical background, the earlier models

The dynamics of the western tropical Atlantic and the NBC retroflection were examined analytically by Csanady (1985, 1990) and Condie (1991) using steady inertial models. Although a portion of the NBC water leaks to the north along the coast in some of these models, the leakage is steady and, as such, it does not explain how and why NBC rings are generated. Also, both the high resolution numerical models used by Philander and Pacanowski (1986) (i.e. the GFDL Tropical Atlantic Model) and the NCAR Community Model (Holland and Bryan, 1987; Schott and Boning, 1991) fail to produce NBC rings (Fratantoni et al, 1995; Johns et al, 1990; Didden and Schott, 1993). In contrast, a modification of the Navy Model [originally developed for the Gulf of Mexico by Hurlburt and Thompson (1980)] does exhibit NBC rings (Fratantoni et al, 1995). Why this is so is

²Note that by “instability” we mean here the breakdown of a known steady solution such as the breakdown of a zonal jet (e.g. the separated Gulf Stream or the Kuroshio). We do not refer to all time-dependent processes as “instabilities”.
not clear. OU and DE RUIJTER (1986) suggest that Agulhas retroflection rings are produced as a result of curvature constraints. Although their model is useful and may shed some light on the separation process, it does not provide information regarding the eddies' generation frequency.

2.3 The recently developed approach

NP modelled the NBC eddy generation process as follows. First, they considered a northwestward flowing boundary current (with density $\rho$) overlying an infinitely deep lower layer [with a density $(\rho + \Delta \rho)$]. It is assumed that the current turns eastward (Fig. 2a) after separating from the wall and that the separation is a result of vanishing wind stress curl over the ocean interior; i.e. the separation is forced by the interior dynamics which is taken to be independent of the boundary currents dynamics. In reality, the two are probably not truly independent but the "isolation approach" is a useful tool for understanding the processes in question. This approach implies that when the current does not retroflect (winter and spring) the curl of the wind stress does not vanish along 7°N; this is in agreement with the observations.

Since the length scale of our highly nonlinear boundary currents system and its retroflection is much smaller than the basin length scale, the direct wind input to it is small compared to the input over the entire basin and, therefore, can be neglected. This means that our current's inertia is much more important than local forcing by the wind. In this sense, our model is very different from PARSON's (1969) linear model where the length scale of the western boundary current is much broader than the Rossby radius so that local wind action is important.

Fig. 2a. Schematic diagram of the retroflected North Brazil Current (NBC). The cause of the retroflection is not important for the present study but one may assume that it results from a vanishing wind stress curl over the interior. The dashed line shows the boundary of the integration area. $\gamma$ is the tilt of the coastline. $H$ is the undisturbed depth away from the wall where the velocity vanishes.

(Reproduced from NP.)
In a fashion similar to the neglect of the local wind stress, local bottom friction can also be neglected. To see this, recall that most of the dissipation of the general oceanic circulation occurs underneath the western boundary currents, because the boundary currents flow over a long distance, comparable to the basin scale. Our western boundary current system is short relative to this length scale and, therefore, bottom friction can also be neglected.

Using the above approximations, NP showed that, even though the scenario of a steady current that turns back on itself seems reasonable, it cannot exist, because the westward flow-force associated with the water entering and leaving the box (bounded by the dashed line shown in Fig. 2a) cannot be balanced. A resolution of the paradox requires the generation of eddies which would leave the box on the left-hand side (looking northward). Under such conditions, the eddies compensate for the flow-force exerted on the box by the flow on the right-hand side. This is shown below in Section 2.4 using a computational method that involves analytical integration of periodic flows over the box (Fig. 2a). A perturbation scheme in a small parameter representing the ratio between the eddy drift speed and the current speed is then applied to simplify the resulting algebraic equations.

2.4 The retroflection paradox

Consider a northwestward boundary current carrying relatively light water (with density \( \rho \)) retroflecting into an otherwise stagnant ocean (with density \( \rho + \Delta \rho \)) and flowing eastward (Fig. 2a). As mentioned, the cause of the retroflection is not important for the present analysis, but one may consider it to be a result of vanishing wind stress over the ocean interior. If one (temporarily) assumes that a steady state is reached then one can integrate the steady nonlinear x momentum equation over the region (S) bounded by the dashed line ABCDEFG shown in Fig. 2a,

\[
\int \int_S \left[ hu \frac{\partial u}{\partial x} + hv \frac{\partial u}{\partial y} - fvh + g'h \frac{\partial h}{\partial x} \right] dxdy = 0.
\]

(2.1)

where, because of the tilt of our coordinate system, the Coriolis parameter is given by \( f = f_0 + \beta (y \cos \gamma - x \sin \gamma) \), \( h \) is the thickness of the light water, \( S \) is the integration area, and the remaining notation is conventional. (Note that, for convenience, the variables are defined both in the text and the Appendix.)

Using the continuity equation and a streamfunction \( \psi \) (defined by \( \partial \psi / \partial x = vh; \partial \psi / \partial y = -uh \)), Eq. 2.1 can also be written as,

\[
\int \int_S \left[ \frac{\partial}{\partial x} (hu^2) + \frac{\partial}{\partial y} (huv) \right] dxdy - \int \int_S f \frac{\partial \psi}{\partial x} dxdy + \frac{g'}{2} \int \int_S \frac{\partial (h^2)}{\partial x} dxdy = 0.
\]

(2.1a)

It is now assumed that the angle \( \gamma \) is small so that \( \cos \gamma = 1 \) and \( \sin \gamma = 0 \) (Fig. 2b) implying that the approaching and retroflecting currents are parallel and that \( \partial (f \psi) / \partial x = f \partial \psi / \partial x \). It will become clear later that, to the order of our approximation, \( ([BR / f_0]^{1/6}) \) this requires that \( \gamma \approx (BR / f_0)^{1/6} \), where \( R_d \) is the Rossby radius. For our problem, \( BR / f_0 \approx 0.2 \) so that the coastline tilt of the South American continent (\( \approx 30^\circ \), see Fig. 1) can indeed be neglected. With the aid of the Stokes's theorem [i.e. \( \int \int_S (\partial M / \partial x + \partial N / \partial y) dxdy = \oint h uv \ dx - \oint \partial (h^2 / 2 - f \psi) dy = 0 \)], (2.1a) can now be simplified to,

\[
\oint h uv \ dx - \oint \partial (h^2 / 2 - f \psi) dy = 0.
\]

(2.1b)

where BCDEFG is the boundary of \( S \).
Defining $\psi = 0$ where $h = 0$ and noting that $v = 0$ on the solid boundary, one finds that (2.1b) can also be written as

$$\int_C^D [hu^2 + g'h^2/2 - f\psi] \, dy = 0 . \quad (2.2)$$

Noting further that since the flow is geostrophic in CD [so that $fu = -g'\partial h/\partial y$ which, upon multiplication by $h$ and integration in $y$ from a point within the current to its edge, gives $f\psi = g'h^2/(2 - \beta \int_y^L \psi \, dy)$, relation Eq.2.2 reduces to the simple relationship,

$$\int_0^L hu^2 \, dy + \beta \int_0^L (\int_y^L \psi \, dy) \, dy = 0 , \quad (2.3)$$

where $L$ is the combined width of the approaching and retroflecting current.

The curious result is that condition Eq.2.3 cannot be satisfied because $y$ and $\psi$ are always positive along CD and the integration is done from small to large $y$. Note also that, for most boundary currents [i.e. $U \sim O(g'H)^{1/4}$; $L \sim O(g'H)^{1/2}$], the second term $[O(\beta L/f_0)]$ is much smaller than the first term $[-O(1)]$. The impossibility of satisfying Eq.2.3 implies that there cannot be a steady state of the kind originally assumed because the integrated momentum (i.e. momentum flux or "flow-force") imparted (by the fluid entering and exiting through CD) on the control volume (bounded by BCDEFG) cannot be balanced. One way that the retroflected
flow field can resolve this paradox and balance the momentum flux is by somehow creating a flow in the opposite sense (i.e. the negative x direction). However, a steady light westward current with a finite cross-section is impossible without a wall on the north side (against which it can lean) which is absent in our configuration. As we shall see, this means that, on a z plane, the balance can be achieved via the generation of anticyclonic eddies which would propagate to the left due to $\beta$ (Fig.3). We shall show then that a steady state cannot exist (see also Marotzke and Wunsch, 1993, who questioned the existence of a steady state for the general oceanic circulation); instead, eddies must be periodically generated. The computation of this

Fig.3. The resolution of the retroflection paradox. Because of the imbalance shown in Fig.2, anticyclonic eddies are generated on the left-hand side (looking offshore). Through the $\beta$ effect these eddies are forced to propagate to the left. Under such conditions the flow-force exerted by the retroflected flow on the right is now balanced by the force corresponding to the eddies' transport to the left. We shall see that, even though the mass transport of the eddies is small compared to the transport of the approaching and retroflecting currents, the flow-force of the drifting eddies is comparable to that of the currents. The "wiggly" arrow denotes migration and should be distinguished from solid arrows which denote flow direction. $L$ is the combined width of the (known) approaching current and the (unknown) retroflecting current. (Reproduced from NP.)

situation is discussed in the following sections.

2.5 The unsteady integrated momentum theorem

As mentioned, to solve for the size, number and periodicity of the generated eddies NP applied the integrated momentum theorem for periodic flows. This general equation enabled them to circumvent the time dependent generation process itself. That is, the technique gave all the needed information regarding the eddies exiting the domain and the outcome of the generation process (without giving information about the detailed formation process itself).

To derive the desired equation NP began with an integration of the time dependent x momentum equation (multiplied by $h$) once in time,

$$\int_0^T h \frac{\partial u}{\partial t} \, dt + \int_0^T h u \frac{\partial u}{\partial x} \, dt + \int_0^T h v \frac{\partial v}{\partial y} \, dt - \int_0^T h f v \, dt = - \int_0^T g' \frac{\partial h}{\partial x} \, dt. \tag{2.4}$$

where $T$ is the period of eddy generation, and, since we took $\gamma$ to be small, $f = f_0 + \beta y$.

Clearly, this is a physical rather than a mathematical paradox. From a strict mathematical point of view the problem is simply over-constrained (by the forced steadiness) rather than paradoxial. However, from a physical point of view one would expect such a steady situation to exist.
Leaving Eq.2.4 aside for a moment, we note that multiplication of the continuity equation by \( u \) and integration in time over one period gives,

\[
\int_0^T u \frac{\partial h}{\partial t} \, dt + \int_0^T u \frac{\partial}{\partial x} (hu) \, dt + \int_0^T u \frac{\partial}{\partial y} (hv) \, dt = 0.
\]

Adding this equation to Eq.2.4 gives,

\[
\int_0^T \frac{\partial}{\partial x} (hu^2) \, dt + \int_0^T \frac{\partial}{\partial y} (huv) \, dt + \int_0^T hfv \, dt + \int_0^T g' \frac{\partial}{\partial x} (h^2) \, dt = 0.
\]

(2.4a)

because \( \int_0^T \frac{\partial}{\partial t} (hu) \, dt = hu \bigg|_T - hu \bigg|_0 = 0 \) (i.e. the system always returns to its original structure after one generation period). We now integrate Eq.2.4a over \( S \) (Fig.3). Since the domain \( S \) is fixed we can exchange the order of integration with respect to time and space. One finds,

\[
\int_0^T \int_S \frac{\partial}{\partial x} (hu^2) \, dx \, dy \, dt + \int_0^T \int_S \frac{\partial}{\partial y} (huv) \, dx \, dy \, dt - \int_0^T \int_S hfv \, dx \, dy \, dt + \int_0^T \int_S \frac{\partial}{\partial x} (h^2) \, dx \, dy \, dt = 0.
\]

(2.5)

Next, we define a time-integrated streamfunction \( \tilde{\psi} \) (to be distinguished from the steady stream-function \( \psi \) which is defined in the usual manner),

\[
\tilde{\psi}_y = - \int_0^T hudt,
\]

\[
\tilde{\psi}_x = \int_0^T hvdt.
\]

(2.6)

This definition stems from an integration of the continuity equation in time (from zero to \( T \), noting again that \( h(T) \) and \( h(0) \) and changing the order of integration so that,

\[
\frac{\partial}{\partial x} \left( \int_0^T hudt \right) + \frac{\partial}{\partial y} \left( \int_0^T hvdt \right) = 0.
\]

(2.7)

The streamfunction and Stokes's theorem (and the fact that along the zonal boundary at least one of the three variables \( h, u \) and \( v \) is always zero), enables one to express Eq.2.5 as,

\[
\int_0^T \int_0 c h^2 \, dy \, dt - \oint_\phi f \tilde{\psi} \, dy + \frac{g'}{2} \oint_\phi \int_0^T h^2 \, dy \, dt = 0,
\]

(2.8)

where the arrowed circle denotes counterclockwise integration and \( \phi \) is the boundary of the integration area \( S \). Assuming that the flow away from the eddy generation region across CD (Fig.3) is steady, Eq.2.8 can also be written as,

\[
\int_0^T \int_G (hu^2 + g'h^2/2) \, dy \, dt - \int_{\text{F}} F \tilde{\psi} \, dy = T \int_C \int_D (hu^2 - f\psi + g'h^2/2) \, dy
\]

(2.9)

where, as before, \( \psi \) is the steady streamfunction defined in the usual manner.

\textit{Equation 2.9 is the desired flow-force balance.} In an analogy to a rocket and a spinning sprinkler, the right-hand side corresponds to the flow-force exerted on the domain by the
water entering and exiting on the right. It is the flow-force (or integrated momentum) that we argued is not balanced unless eddies are shed on the left. The term on the left-hand side of Eq. 2.9 corresponds to the flow-force exerted by the eddies moving to the left out of the domain. In this context, it is important to point out that the role of $\beta$ in our problem is to arrest the growth of the eddies and remove them from the generation area. It is a simple matter to compute the contribution of the steady current on the right to Eq. 2.9, but the computation of the eddies' contribution (which is time-dependent) is not so trivial. It is, therefore, discussed in detail in the next section.

2.6 The transformed integrated momentum balance

As mentioned, the use of Eq. 2.9 is not straightforward because all the terms on the left-hand side are time-dependent. To simplify the procedure, NP transformed those terms to a new coordinate system $(\hat{x}, \hat{y}, \hat{t})$ moving steadily westward at the speed of the eddies. After much algebra they derived the expression,

$$
\begin{align*}
&- C \int_{h_1}^{h_2} \int_0^a h d\hat{x} d\hat{y} + \int_{h_1}^{h_2} \left( \int_0^a h d\hat{x} \right) d\hat{y} d\hat{t} = T \int_0^L \int_0^L h u d\hat{y} d\hat{t} + \beta T \int_0^L \int_0^L \psi d\hat{y} d\hat{t}.
\end{align*}
$$

(2.10)

where $C$ is the eddy migration rate (measured positively eastward), $b_1$ and $b_2$ correspond to the southernmost and northernmost latitudes of the existing eddy and zero and $a$ to the westernmost and easternmost longitudes (as viewed from the moving coordinate system). The left-hand side of Eq. 2.10 corresponds to the momentum flux (or flow-force) exerted on the region by the exiting eddies whereas the right-hand side is associated with the momentum flux of the steady approaching and retroreflecting currents. By noting that the eddies' migration rate is $O(\beta R_d^2)$ so that $C/f_d R_d^2 \approx O(\beta R_d^2/f_d) \ll 1$ and $\beta R_d^2/f_d \ll 1$ (where $R_d$ is the eddy and approaching current Rossby radii), it is a simple matter to show that the dominant terms in Eq. 2.10 are I and II (see NP). This point will be further discussed in Section 2.8.

Four comments should be made with regard to Eq. 2.10. First, it is important to realize that, even though Eq. 2.10 represents a balance of momentum flux integrated over a period $T$, the time-dependent momentum flux varies as each eddy passes (through the western boundary) leaving an instantaneous and periodic imbalance. This means that the western front of the flow also cannot be steady. It must advance and retreat into the control volume in accordance with the eddy formation process. A similar aspect can be noted in the observations of Richardson et al (1994). Second, it is important to realize that not all situations corresponding to Eq. 2.10 are associated with the generation of eddies. This results from the fact that, in the absence of an eddy removal mechanism (e.g. $\beta = 0$), the flow may not be periodic but rather may evolve constantly in time.

Third, Eq. 2.10 and the other familiar constraints (e.g. potential vorticity and conservation of mass) are not sufficient to close the problem. A closure condition is necessary and it is assumed here that the eddies are in contact with each other as they leave the domain. Namely, although spacing between the eddies is clearly possible, one has to make some sort of an assumption about the exiting eddies in order to close the problem. It is difficult to say where the edge of actual eddies in the ocean is but the assumption that the eddies are in contact with each other appears to be in reasonable agreement with observations (Richardson et al 1994). Even if it were not in good agreement it would have still been very useful because it provides
an upper bound on the generation frequency and eddy size. Fourth, another way of looking at the eddy generation process is by noting that, since any meridional flow has a tendency to drift westward due to $\beta$ (see, e.g. NOF and DEWAR, 1993, and the references mentioned therein), section AB (Fig. 2b) also has a tendency to drift to the west. This means that $\beta$ tends to tear the western portion of the retroflection apart from the main flow.

2.7 Other constraints

In addition to the above momentum constraint, the field must, of course, satisfy the continuity equation,

$$- \int_C \int_{\text{eddy}} h u dy + \frac{1}{T} \int \int_{\text{eddy}} h dx dy = 0$$

and the potential vorticity equation,

$$\frac{D}{Dt} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + \gamma \right) = 0$$

NP have shown that the above system of equations (Eqs 2.9 - 2.12) enables one to close the problem provided that we assume that the resulting eddies are in contact with each other as they leave the domain. (Note that this does not mean that, during the formation process, the current is always in contact with the separated ring. All it says is that the eddies touch each other when they leave the generation area.) Since the approaching current is known and the $\beta$-induced eddy drift is also known (see e.g. NOF, 1981), Eq. 2.10 involves two unknowns (the retroflected current transport and the eddies’ size). The solution for these two unknowns will be provided by Eqs 2.11 and 2.12.

2.8 Scales

Since the eddies are slowly removed from the generation area by $\beta$ (at a speed of the order of $\beta R_{de}^2$) one expects their length scale to be much greater than the current’s length scale. Indeed, a comparison of the second term on the left and the first term on the right of Eq. 2.10 (i.e. the largest term on each side) gives,

$$f H_e R_{de}^3 \sim O \left[ \frac{R_{de}}{\beta R_{de}^2} H (g'H) R_d \right]$$

which yields

$$R_{de} \sim O \left( R_d / e^{1/6} \right),$$

(2.13)

where $\varepsilon \equiv \beta R / f_0 \ll 1$, and it has been assumed that the eddies moving out of the area are not far from each other [i.e. $T \sim O(a/C)$]. (As mentioned, NP assumed that the generated eddies are in contact with each other as they leave the control box but it is not necessary to make this assumption at this stage.) Note that in obtaining Eq. 2.13 it has been taken into account that, because of the definition of the Rossby radius $[(g'H)^{1/2}/f]$, the ratio of the eddy depth scale, $H_e$, and the approaching current depth scale, $H$, is given by $R_{de}^2 / R_d^2$. In view of Eq. 2.13, the periodicity at which the eddies are generated ($T$) is,
These scales are sensible for the NBC as the parameter $\epsilon$ is $\sim O(0.1)$ implying that the eddies are perhaps twice or $1\frac{1}{2}$ times as large as most rings; the eddies' depth scale $h_e$ is of the order of the current depth divided by $\epsilon^{1/2}$.

Given the above scales, we now introduce the following nondimensional parameters. For the currents entering and leaving the control volume through CD, the scaled variables are

$$x^* = x/R_d; \quad y^* = y/R_d; \quad v^* = v/(g'H)^{1/2}$$

$$h^* = h/H; \quad R_d = (g'H)^{1/2}/f_0; \quad \psi^* = \psi/g'H^2/f_0$$

Similarly, the scaled variables for the eddies are,

$$x_e^* = x/R_{de}; \quad y_e^* = y/R_{de}; \quad h_e^* = \frac{h}{H}\left(\frac{R_{de}}{R_d}\right)^2$$

$$C^* = C/\beta R_{de}; \quad R_{de} = \alpha R_d/\epsilon^{1/6}; \quad \alpha \sim O(1)$$

$$\epsilon = \beta R_d/f_0 \ll 1; \quad T^* = \frac{T}{\epsilon^{1/6}/\beta R_d}$$

where $C^*$ is the nondimensional eddy migration rate, $\alpha$ is an unknown number of order unity that measures the size of the resulting eddies and the subscript "e" indicates association with the eddies. Note that the introduction of $\alpha$ is necessary because the Rossby radius of the eddies determines their size and speed and that the above scaling is the only one that satisfies both the momentum and mass balance.

2.9 The nondimensional equations

Substitution of the above nondimensional variables into the integrated momentum gives,

$$C^* \alpha^6 \epsilon^{1/6} \int_{b_1}^{b_2} \int_{y_e}^{a_e} h_e^* dx_e dy_e^* + \alpha^5 \int_{y_e}^{a_e} (1 + \alpha^5 \epsilon \psi_e^*) \int_{y_e}^{a_e} \left( \int_{y_e}^{a_e} h_e^* dx_e^* \right) dy_e^*$$

$$= T^* \int_{0}^{L_e^*} h_e^* (u_e^*)^{2} dy_e^* + \epsilon T^* \int_{0}^{L_e^*} \int_{y_e^*}^{L_e^*} \psi_e^* dy_e^* dy_e^*$$

where $L_e^*$ is the nondimensional combined width of the approaching and retroreflecting currents, $b_1^*$ and $b_2^*$ are the nondimensional distances between the eddy edges and the coast, and $a^*$ is the nondimensional eddy width along the $x$ axis. Similarly, the continuity equation can be written as,

$$\int_{0}^{L_e^*} h_e^* u_e^* dy_e^* + \frac{\alpha^4 \epsilon^{1/6}}{T^*} \int_{0}^{a_e^*} \int_{b_1}^{b_2} h_e^* dy_e^* dx_e^* = 0.$$
with the observations of Richardson et al., 1994) and the fact that we know how fast eddies migrate on a β plane, will enable us to close the problem. To obtain the solution, all the dependent variables are now expanded in a power series in ε^{1/6} and only the highest-order terms are kept. This enables one to obtain the solution. Note that an epsilon of 0.10 corresponds theoretically to a possible error which is greater than 50% because the expansion is done in powers of ε^{1/6}. While such a relatively large value for ε is certainly not desired, it is in line with other frequently used approximations such as the quasi-geostrophic approximation. Despite the fact that the latter approximation is based on a small amplitude expansion, it is routinely applied to eddies and rings whose amplitudes are sometimes even larger than the mean depth (i.e., ε > 1!). And yet, it frequently gives the correct answer because the components of the neglected terms which do not contain ε are evidently small. That is to say, perturbation schemes often work even if ε is relatively large and it is hard to tell in advance how small ε should be.

2.10 Solution

Since our scaling implies, and the observations (Fratantoni et al., 1995) confirm, that the eddies are much deeper than the parent current [\(H_e \sim O(H/ε^{1/3})\)], it follows that the parent current must have strong anticyclonic vorticity. This results from conservation of potential vorticity. It implies that the eddies' vorticity will be anticyclonic (despite the severe stretching of the water column which occurs during the formation) only if the parent current relative vorticity is strongly anticyclonic (i.e. close to -f). The NBC satisfies this condition of strong anticyclonic vorticity; observations suggest that its potential vorticity is indeed close to zero (see e.g. Csanady, 1985). What would happen when the parent current relative vorticity is not strongly anticyclonic is not entirely clear. We suspect, however, that since the eddy generation period is long, even a small amount of horizontal friction within the core of the growing eddy can alter its original cyclonic vorticity to anticyclonic vorticity. Numerical experiments of a somewhat related case (where an outflow of a northward channel generates eddies) shows this kind of vorticity generation (Pichevin and Nof, 1996). Note, however, that boundary frictional effects will most likely generate positive rather than negative relative vorticity, but these effects are probably confined to a thin layer along the left edge of the current. As a result, this effect will only be noticed along the edge of the rings where the observed relative vorticity is frequently cyclonic. A detailed investigation of this aspect is beyond the scope of this study.

2.10.1 Zero potential vorticity eddies. NP first looked at the case where both the current and rings have zero potential vorticity. For this case the solutions for both the current and the eddies are straightforward despite the nonlinearity. Specifically, the leading-order solution for the approaching and retroflecting current is,

\[
\begin{align*}
u &= f_0 \left( Y - \frac{L}{2} \right) \\
h &= -\frac{f_0^2}{2g} (Y^2 - Ly) 
\end{align*}
\]

where L is the (combined) currents' width given by,

\[
L = 2(2gH)^{1/2}/f_0
\]

and we have already taken the zeroth-order balance (discussed in NP) into account in the sense
that we forced the flux of the retroflected current to be equal to that of the approaching current. Note that this zeroth-order balance does not mean that the eddies are neglected. It means that, although the contribution of the eddies to the momentum balance is of $O(1)$, their contribution to the overall mass balance is much smaller $[O(\epsilon^{1/6})]$ and, consequently, can be neglected to zeroth-order.

Similarly, the leading-order solution for the eddy is,

$$v_0 = -\frac{f\ell}{2}, \quad h = \frac{f^2}{8g} (8R_{de}^2 - r^2)^{1/6},$$

(2.17)

where, for simplicity, we use here a polar coordinate system $(r, \theta)$ whose origin coincides with the center of the eddy; $R_{de}$ is the (unknown) momentum Rossby radius of the eddy. Substitution of Eqs 2.15, 2.16 and 2.17 into the zeroth-order momentum equation gives one equation with one unknown, the eddy size $(R)$ which is related to its Rossby radius $R_{de}$.

One finds,

$$\frac{21/4}{I} \frac{24(5\pi)^{1/6}}{27/4\ell^{1/6}} R_{de}^{2/3} = \frac{27/4}{I} \frac{24(5\pi)^{1/6}}{27/4\ell^{1/6}} R_{de}^{2/3}$$

(2.18)

Note that in deriving the solution it has been taken into account that the westward migration of a zero potential vorticity eddy resulting from $13$ is $-\frac{\pi}{4}R_{de}$ (see NOF, 1981).

2.10.2Finite potential vorticity eddies. NP also looked at the case where the parent current potential vorticity is still zero (or very close to zero) but the eddies' (uniform) potential vorticity is nonzero and finite. At first, this appears to be contradictory to the idea of potential vorticity conservation, but a close examination reveals that it is consistent with our scaling and expansion. All it means is that, with our approximation, there can be a difference of $O(\epsilon^{1/6})$ between the two potential vorticities (parent current and eddies). When this difference is combined with our scaling one can then see that, because the eddies are much deeper than the current, the parent current potential vorticity must always be taken as zero (regardless of the eddies' potential vorticity). To see this, recall that

$$\bar{\xi} + f = \frac{\xi + f}{h_c} = \frac{h}{H_p},$$

where $\xi$ and $\xi_c$ are the relative vorticities of the current and eddy, $h$ and $h_c$ are their depths and $H_p$ is the "potential vorticity depth" of both the current and the eddies. We now note that, according to our scaling, $h \sim O(\epsilon^{1/3}H_p)$ and $h_c \sim O(H_p)$. This means that even though the eddies may have uniform and finite potential vorticity $[h_c \sim O(H_p)]$, the current effectively (i.e. to zeroth-order) has zero potential vorticity because its depth $(h)$ is much smaller than the potential vorticity depth, i.e. $\frac{\xi + f}{h} = h(h/H_p) \sim O(\epsilon^{1/6})$.

For the case of finite potential vorticity eddies the final calculations must be done numerically. Such calculations were performed for the case where the eddies' relative vorticity at the center varies from $-f_0$ to zero (Fig.4). Note that the eddy $\beta$-induced westward speed was calculated from the formula given by NOF (1981) and the structure of the uniform potential
Fig. 4. The eddy radius (R*), generation period (T*), drift speed (C*), and orbital speed along the edge (vθ*) as a function of the eddy's relative vorticity (ξe) at the center for the no-topography problem. Zero potential vorticity corresponds to ξe/fo = -1; all other values correspond to positive uniform potential vorticity.
vorticity eddies from the relationships given by FLIERL (1979). Note also that the main
difference between the zero and finite potential vorticity cases is that, in the finite potential
vorticity case, both the periodicity and the eddy size are larger than those of the zero potential
vorticity.

2.11 Application

Typical numerical values for the NBC are $Q = 45 \text{ Sv}$; $f_0 = 2 \times 10^{-5} \text{s}^{-1}$; $g' = 2 \times 10^{-2} \text{m} \text{s}^{-2}$ and
$\xi/f_0 \approx -0.6$ (where $Q$ is the approaching current transport). The above coriolis parameter
corresponds to a latitude of $8^\circ \text{N}$, and the above relative vorticity, $(\xi/f_0 = -0.6)$ corresponds
to the observed core of many NBC rings (see e.g. RICHARDSON et al, 1994). For these values,
our calculated upstream depth $H$, Rossby radius $R_d$ and $\varepsilon = \beta R_d/f_0$ are $300\text{m}$, $122\text{km}$ and
$0.122$, respectively.

Our corresponding solution (Fig.4) indicates that $R^* \approx 2.2$ and $T^* \approx 16$ so that the resulting
eddies' radius is approximately $380\text{km}$ and the generation period is roughly $55$ days. These
reasonable values correspond to a migration rate that is a bit too high ($18\text{cm s}^{-1}$ compared to
the observed value of $10\text{cm s}^{-1}$ or so (RICHARDSON et al, 1994; FRATANTONI et al, 1995)) and
an eddy's volume flux that is also high ($16\text{ Sv}$ compared to a temporal average (i.e. an average
for the fraction of the year during which the NBC retroreflects) of roughly $8\text{ Sv}$ (FRATANTONI
et al, 1995)). Although these values are reasonable for a process-oriented study such as ours,
a still better estimation can be made by taking into account that the actual migration rate of
the eddies is reduced as a result of the topography. This is consistent with FRATANTONI et al's
(1995) conclusion that NBC rings migrate at half the rate of their open ocean counterparts.
The details of the corresponding computations are given in NP and need not be repeated here.

When the effect of topography is included the westward migration is reduced. How much
it is reduced is not entirely clear as it is not obvious what fraction of the eddy (if any) is in
contact with the bottom. However, the fact that NBC eddies are migrating to the northwest
(parallel to the coastline) rather than toward the southwest (which is their uninterrupted open
ocean propagation pattern) strongly suggests that they interact with the topography. On the
basis of Fratantoni et al's (1995) analysis, NP took the observed speed of $10\text{cm s}^{-1}$ to be a
result of a speed reduction caused by topography.

For an NBC transport of $45\text{ Sv}$ such a reduced northwestward speed gives a periodicity of
$110$ days, a radius of $463\text{ km}$ and a transport of $17.4\text{Sv}$. A reduced speed of $12\text{cm s}^{-1}$ gives,
on the other hand, a somewhat shorter periodicity of $87$ days, a smaller radius of $445\text{km}$ and
a somewhat larger transport of $18.9\text{Sv}$. All of these values, save the transports, are more
reasonable than the previous ones which neglected bottom topography. The transport of $17$-
$19\text{Sv}$ (corresponding to an annualized transport of $8-9\text{Sv}$), which is about a third of the
upstream transport, is, no doubt, relatively large compared to the observed annualized value
of $4\text{Sv}$. This is probably a result of our relatively large $\varepsilon (-0.12)$ which corresponds to an error
of more than $50\%$ either because the expansion is done in powers of $\varepsilon^{1/6}$, or is a result of the
difficult in identifying the edge of the actual NBC rings.

An additional difficulty is that in all of the above applications the predicted speeds along
the edge of the rings are about $3\text{m s}^{-1}$, a value that is obviously too high. This is unavoidable
(and typical) for reduced gravity models; it results from the high amplitude along the edge (see e.g. FLIERL, 1979). Our predicted radii also appear to be large but this is again a result of the reduced gravity model and our use of a lens model for the eddies.

The above results are not very sensitive to the choice of the NBC transport. For instance, with a transport of 25Sv (instead of the 45Sv discussed above) and no bottom topography, the radius of the predicted eddies is 340km (instead of 380km) and the periodicity is 60 days (instead of 55 days). (The corresponding values for the undisturbed depth, Rossby radius and epsilon are 225m, 106km and 0.11 respectively.) With a bottom topography which causes a migration rate of 10cm s⁻¹, the predicted radius is 385km and the periodicity is 71 days. This implies that the topographic case is more sensitive to the transport than the non-topographic case.

3. THE WESTERN EQUATORIAL PACIFIC

We shall begin this section with a description of the surface currents system in the western tropical Pacific. Using recently developed models, we shall then demonstrate that the absence of constantly shed rings in the Pacific is mainly a consequence of the southward flowing Mindanao Current which does not have a counterpart in the Atlantic.

3.1 Observational background

The western equatorial Pacific plays a major role in the establishment of El Niño and Southern Oscillation (e.g. WEBSTER and LUKAS, 1992). Because of the Pacific-to-Indian throughflow, it may also be an important part of the so-called great-conveyor circulation (e.g. GORDON, 1986). The important elements of these processes are surface currents and it is for this reason that the upper waters are the focus of this section.

The upper circulation consists of two western boundary currents that flow toward the equator and provide closure to two Sverdrup gyres (Fig.1b). The two gyres are not symmetrical relative to the equator (KESSLER and TAFT, 1987). One is situated solely in the Northern Hemisphere whereas the other contains the equator (although most of it is in the Southern Hemisphere). The eastward flowing North Equatorial Counter Current (NECC) forms the boundary between these two gyres at about 5°N. North of the NECC the westward flowing North Equatorial Current (NEC) bifurcates as it encounters the Philippines (TOOLE, MILLARD, WANG and PU, 1990). One branch flows northward to form the Kuroshio and the other flows southward to form the MC. A similar situation exists in the Southern Hemisphere where the westward flowing South Equatorial Current (SEC) bifurcates at about 15°S, with one branch flowing northwestward and the other southward. The northwestward branch of the SEC reverses direction during the northwest monsoon (see, e.g. FINE, LUKAS, BINGHAM, WARNER and GAMMON, 1994; SCHOTT, 1939).

The physical characteristics of the above currents are as follows. The MC extends to a depth of 600m (WYRTKI, 1961) and a distance of 100km offshore. Estimates of its transport range from 8 to 35Sv (WYRTKI, 1961; CANNON, 1970; NITANI, 1972; TOOLE, ZOU and MILLARD, 1988; LUKAS, FIRING, HACKER, RICHARDSON, COLLINS, FINE and GAMMON, 1991). The SEC transports about 40Sv westward (KESSLER and TAFT, 1987). Of this amount, 8-14Sv flows northwestward through the Vitiaz Strait which is situated immediately to the east of Papua New Guinea at approximately 148°E and 6°S (LINDSTROM, BUTT, LUKAS and GODFREY, 1990). In addition to this, a few Sverdrups flow northwestward through the St George's Channel which is situated about 500km to the east of the Vitiaz Strait at approximately 152°E and 4°S (BUTT and LINDSTROM, 1994).
The convergence of the SEC northward branch and the southward flowing MC lead to the eastward flowing NECC which transports roughly 35Sv (Kessler and Taft, 1987) and the southwestward flowing Indonesian Throughflow which carries around 15Sv or less (see e.g. Wyrtki, 1961; Piola and Gordon, 1984; Fine, 1985; Fu, 1986; Toole et al., 1988; Murray and Arief, 1988; Murray, Arief, Kindile and Hurlburt, 1990; Cresswell, Frische, Peterson and Quadsafel, 1993; Wijffels, 1993; Fieux, Andrie, Delecluse, Ilahude, Kartavtseff, Mantisi, Molcard and Swallow, 1994; Meyers, Bailey and Worby, 1995). For instance, Fieux et al. (1994) calculate a transport of 18.6 ± 7Sv whereas Piola and Gordon (1984) suggest a 10-14Sv range. There are two semi-permanent eddies north and south of the convergence zone. The first, the Halmahara eddy, is situated south of the NECC and has an anticyclonic circulation, whereas the second, the Mindanao eddy, which is situated north of the NECC, has a cyclonic circulation (Wyrtki, 1961). Unlike their Atlantic counterparts, these eddies are not shed on a regular basis. For related aspects of the western equatorial Pacific the reader is referred to Fine et al. (1994) and Wijffels, Toole, Bryden, Fine, Jenkins and Bullister (1996).

3.2 Theoretical background

None of the earlier studies focused directly on the convergence in the western Pacific. There have been a number of related numerical simulations which have focused either on the Indonesian Throughflow or the general circulation in the region (e.g. Cox, 1975; Godfrey and Golding, 1981; Kindile, Hearn and Rhodes, 1987; Kindile, Hurlburt and Metzger, 1989; Semtner and Chervin, 1988, 1992; Godfrey, 1989; Masumoto and Yamagata, 1992; Godfrey, Hirst and Wilkin, 1993; Hirst and Godfrey, 1993; Inoue and Welsh, 1993). Although some of these studies display eddies which migrate through the area, none have shown generation and shedding similar to that occurring in the Atlantic.

3.3 Recent approach to the convergence problem

AGRA and NOF (1993; hereafter referred to as AN) and LEBEDEV and NOF (1996, 1997) considered a retroflection associated with an opposing current (i.e. an independent current that approaches the retroreflecting current from the other side as shown in Figs 5 and 6). Their results show that, in this case, the retroflection paradox discussed earlier (in Section 2) does not exist. Under such conditions, the momentum flux of the approaching current is balanced by the momentum flux of the opposing flow, and consequently there is no need for the current to generate eddies. This is easily demonstrated with the AN solution which is briefly presented below. First, it is noted that, in this case, Eq. 2.1b leads to,

\[ \int_A^B h v^2 \, dx + \sin \theta \int_F^E h(\hat{v})^2 \, dx + \int_D^C h v^2 \, dx = 0 \]

rather than to the paradoxical relationship (Eq. 2.3). Here, \( \hat{x} \) and \( \hat{y} \) denote the tilted coordinate system (see Fig. 5) and \( \theta \) is the angle (measured clockwise) formed by the normal to the coast and the \( x \) axis. For simplicity, we have neglected here the terms due to \( \beta \) keeping in mind that the largest terms are of order \( -O(h u^2 dy) \) which are much larger than those caused by \( \beta \). AN have shown that, ultimately, Eq. 3.1 gives,

\[ \sin \theta = \frac{M_1 - M_2}{M_1 + M_2} \]

(3.2)
Fig. 5. Schematic diagram of the stationary collision. Near-wall velocities and depths of the colliding currents are equal in accordance with the AGRA and NOF (1993) solution. The separated current is straight and leaves the coast at an angle $\theta$. The origin is situated at the intersection of the coastline with the straight offshore separating streamline.

Fig. 6. A three-dimensional view of the colliding western boundary current. The opposing jet has a density identical to that of the approaching western boundary current. Most currents in the ocean are expected to be unbalanced so that the system would drift along the coast at speed $C$ (positive for a migration toward the equator). The "wiggly" arrow denotes migration and should be distinguished from solid arrows which denote actual speeds. Adapted from AGRA and NOF (1993).
where
\[ M_1 = \int_0^\infty [h v_r^2] y \to -\infty \, dx \] (3.3)
is the momentum flux of the approaching current, and
\[ M_2 = \int_0^\infty [h v_r^2] y \to +\infty \, dx \] (3.4)
is the momentum flux of the counter current. This completes the presentation of the AN solution.

The difficulty with the above solution is that the condition of stationarity associated with the derivation of this solution requires a particular relationship between the approaching and opposing current. Most oceanic currents will probably not satisfy this special relationship. To demonstrate this we note that the approaching current is geostrophic and has a transport of
\[ T_1 = g \left( H_1 - H_1w \right) \] where \( H_1 \) is the offshore undisturbed depth of the poleward flowing approaching current and \( H_1w \) is its depth near the wall; \( H_2 \) is the offshore undisturbed depth of the counter current, and \( H_2w \) is the opposing current near-wall depth. Similarly, the transport of the counter current is
\[ T_2 = g \left( H_2 - H_2w \right) \] and a straight retroflecting current (NECC) has a transport of
\[ T_3 = g \left( H_1w - H_2w \right) \] in the stationary state, \((T_1 + T_2)\) must be equal to the transport of the retroflecting current \((T_3)\), implying that \( H_1w = H_2w \) which is the AN criteria of “balanced” currents. By virtue of the Bernoulli principle, one finds that the near-wall velocities must also be equal, i.e. \( v_{1w} = v_{2w} \). The above formulae state that, while the transports of the individual boundary currents can vary (because of the wall), the geostrophic transport of the retroflecting current is fixed (and depends only on the constant undisturbed upper layer offshore depths \( H_1 \) and \( H_2 \)). Therefore, when the total transport of boundary currents differs from the value \( g \left( H_1w - H_2w \right) \), a non-stationary flow must take place.

Using the integrated equations in a moving coordinate system, LN derived the migration rate of such an unbalanced confluence zone. For an opposing current intruding into an approaching current (i.e. \( H_1w > H_2w \)) the migration rate is approximately,
\[ C = \left( g' \right)^2 (H_1w^2 - H_2w^2)^{1/2} 16fM_2 (H_1 - H_2) \] (3.5)
where, as before, \( M_2 \) is defined by Eq.3.4. Relation Eq.3.5 gives a speed that is of the order of Kelvin wave propagation speed or smaller.

An example of how the boundary between the approaching and opposing current (i.e. the potential vorticity front) migrates is shown in Fig.7. The line \( t = 0 \) corresponds to a stationary situation which is later perturbed so that \( H_2w \) obeys \((H_1w - H_2w)/H_1 = 0.1\). As pointed out by LN, the above scenario is probably applicable to the South Atlantic Confluence Zone which migrates along the coast as much as 800km in a season (i.e. a few centimeters per second). However, as we shall shortly see, application of this theory to the western Pacific is not so simple.

### 3.4 Application of the confluence theory to the western Pacific assuming that the Indonesian Passages are closed

Although the situation with the Indonesian Passages closed is obviously not physically realistic, it provides useful information and insight into the processes in question because it
Fig. 7. Positions of the potential vorticity front (i.e. the line separating the approaching and opposing currents) after a step-like increase of the counter current's transport. For these calculations, \((H_{1w} - H_{2w})/H_1 = 0.1; f_0 = 10^4 \text{ sec}^{-1}; g' = 2 \times 10^2 \text{ m s}^{-2}; H_2 = 0\). (Reproduced from LEBEDEV and NOF, 1997).

is readily understood. After understanding this particular case, it will be easier for us to understand the more complex situation with the passages open. From our previous discussion, it is clear that an opposing current can, in general, prevent the formation of retroflection rings and eddies. It is not a priori obvious, however, what happens when the convergence theory of AN and LN is applied to the western Pacific. In particular, it is not clear whether or not the confluence would be stationary with the passages closed. We shall show below that, in fact, the confluence could not be stationary unless the passages are open. Under the latter conditions, the confluence can leak excess water to the Indian Ocean and become stationary (Section 3.5).

3.4.1 General derivations of the depths and speeds north and south of the confluence. Before presenting these calculations, it is recalled that the actual measured depths and speeds cannot be used for our computations because the actual passages are open so that they do not
correspond to our conceptual closed passages situation. Consequently, it is necessary to devise some sort of method to compute those variables. This was done by NOF (1995) in connection with an independent issue of transports through the Indonesian Passages. It is, however, also relevant to the issue at hand and, therefore, is presented below.

The sea-level difference across the Pacific with the passages closed can be easily computed from the familiar vertically integrated x momentum equation,

\[ -\Gamma V = -\frac{g'}{2} \frac{\partial}{\partial x} (h^2) + \frac{\tau_{s}(x)}{\rho}, \quad (3.6) \]

where \( V \) is the (northward) vertically integrated transport (i.e. in the y direction) and \( \tau_{s}(x) \) is the surface wind stress in the x direction (i.e. eastward). Equation (3.6) holds both in the sluggish ocean interior away from the boundaries and in the intense western boundary current where the flow is geostrophic in the cross-stream direction. Within the western boundary current the balance is between the first and second term (i.e. the wind stress is small compared to the Coriolis and the pressure term). In the interior, on the other hand, all three terms are of the same order.

Note that, in this model, energy is supplied by the wind over the entire ocean and is dissipated through interfacial friction within the limits of the western boundary current. This bottom friction is not present in Eq.3.6 because it is directed along the mean meridional flow. Integration of Eq.3.6 from the western to the eastern boundary gives the desired (square of the) sea-level difference,

\[ h^2_{\text{east}} = \frac{2}{g'} \int_{0}^{L_{p}} \frac{\tau_{s}(x)}{\rho} \, dx, \quad (3.7) \]

where \( L_{p} \) is the Pacific basin's length, and it has been assumed that, as long as the Indonesian Passages are closed, there is no net transport within the cross-section (i.e. the boundary current transport cancels the Sverdrup transport). We shall see later that, after the passages are open, the boundary current transport no longer cancels the interior Sverdrup transport.

Traditionally, the upper layer depth along the eastern boundaries is taken to be known because this is where the information is emanating from. Following the observations of COLIN, HENIN, HISARD and OUDOT (1971), we shall take the eastern depth in the Pacific to be zero. In view of the above considerations, Eq.3.7 gives for the Pacific western wall depth,

\[ h_{w} = \left[ -\frac{2}{g'} \int_{0}^{L_{p}} \frac{\tau_{s}(x)}{\rho} \, dx \right]^{1/2}, \quad (3.7a) \]

where the subscript "w" denotes that the variable in question is associated with the wall.

The associated depth at the eastern edge of the boundary current can be estimated from the curl of the wind stress. To do so, consider the linearized y momentum equation,

\[ -\Gamma U = -\frac{g'}{2} \frac{\partial}{\partial y} (h^2) - FV, \quad (3.8) \]

where \( F \) is the coefficient of interfacial friction which, as we shall see, need not be specified for our computations. Elimination of the pressure term between Eq.3.6 and 3.8 and consideration of \( \partial U/\partial x + \partial V/\partial y = 0 \) gives,

\[ \beta V = \frac{1}{\rho} \frac{\partial \tau_{s}(x)}{\partial y} - FV_{x}. \]
which, for the ocean interior, reduces to the familiar Sverdrup transport,

$$\beta V = -\frac{1}{\rho} \frac{\partial \tau_s^{(x)}}{\partial y}$$

(3.8a)

Substitution of $V$ from Eq.3.8a into Eq.3.6 and integration from the eastern edge of the boundary current to the eastern boundary of the Pacific gives an expression for the Pacific offshore depth at the edge of the boundary current $H$,

$$H = \left( \frac{2}{g'} \right)^{1/2} \left[ \frac{f_0}{\beta \rho} \int_0^{L_p} \frac{\partial \tau_s^{(x)}}{\partial y} \, dx - \int_0^{L_p} \frac{\tau_s^{(x)}}{\rho} \, dx \right]^{1/2}$$

(3.9)

Note that, in deriving this expression, it has been again taken into account that the depth along the Pacific eastern boundary vanishes and that the boundary layer width is much smaller than the basin width $L_p$.

To compute the near-wall speeds, the above wind-driven western boundary current will be approximated by an inertial boundary current with uniform potential vorticity. The logic behind this somewhat crude approximation is that, within the immediate vicinity of the confluence, friction and wind stress are unimportant. There are many ways by which such an inertial current can be fitted to the simplified wind-driven model. For the purpose of our modeling efforts it is best to fit the two wind-driven depths along the western and eastern edges of the boundary current (i.e. $h_w$ and $H$ given by Eq.3.7a and Eq.3.9) to the inertial near-wall and off-wall depths. Under such conditions, the transport of the inertial boundary current will be identical to that of the wind-driven current because the geostrophic transport $g'(H^2 - h_w^2)/2f_0$ is independent of the depth distribution within the current. Note, however, that, since the wind-driven and the inertial currents are governed by different sets of differential equations, it is impossible to match all the variables of the two currents.

It is assumed now that, within the inertial boundary current (which approximates the actual wind driven current in the Pacific), the potential vorticity is uniform and equals $f_0/H$, where $H$ is the undisturbed offshore depth away from the wall. Such an approximation is common, both for mid-latitude and cross-equatorial flows. While it is certainly not ideal (because of the proximity to the equator), as a first approximation it is adequate. For a (geostrophic) uniform potential vorticity western boundary current, the velocity and depth fields are,

$$v = v_w e^{x/R_d}$$

$$h = H \left( 1 - \frac{v_w}{f_0} \frac{R_d}{e^{x/R_d}} \right)$$

(3.10)  (3.11)

where the Rossby radius, $R_d$, is given by $R_d = (g'H)^{1/2}/f_0$. As before, the subscript "w" corresponds to the wall. For $x = 0$ (i.e. the western wall), Eq.3.11 gives,

$$v_w = f_0 R_d \left[ 1 - h_w/H \right]$$

(3.12)

which will be used later to obtain the near-wall speed.

Note that in these calculations of the fields north and south of the confluence we take the boundaries to be meridional even though later we shall consider a slanted coastline. This is permissible because the deviations of the coastline from a meridional line are of $O(R_d)$ which is much smaller than the width of the basin along which the wind is acting. It is, of course, possible to do the calculations with a slanted coastline by integrating first along a line normal
to the coast up to a distance of $O(R_d)$ away from the coast and then integrate zonally to the eastern boundary. However, these calculations are basically the same as above; so they do not provide any new insights and, therefore, are not presented.

3.4.2 Computation of the depth and speed south of the confluence. Figure 1 suggests that the region corresponding to our conceptual southern area is situated at approximately 2°N (corresponding to $f_0 = 0.5 \times 10^{-5} s^{-1}$). The annual average wind stress across the Pacific to the east is roughly 0.4 dyn/cm$^2$ and the average wind stress curl is approximately zero (STIRCHERZ, O'BRIEN and LEGLER, 1992; HELLERMAN and ROSENSTEIN, 1983). Taking the length of the Pacific to be about 16,000km we find from Eq.3.7a and 3.9 that $h_w = H = 253$m. Relation Eq.3.12 shows that, for zero wind stress curl ($h_w - H$), the initial near-wall speed $v_w$ is also zero. This means that, with the passages closed, there is actually no northward flowing SEC at 2°N. We shall see later, however, that once the passages are open, a cross-equatorial SEC is established.

3.4.3 The northern area. The region corresponding to our conceptual northern area is situated at approximately 5°N (corresponding to $f_0 = 1.3 \times 10^{-5} s^{-1}$). At this latitude the annual average wind stress across the Pacific to the east is somewhat higher than that at 2°N and is estimated to be about 0.5 dyn/cm$^2$. The average wind stress curl is roughly $5 \times 10^{-9}$ dyn/cm$^3$ (STIRCHERZ et al, 1992; Hellerman and Rosenstein, 1983). For these variables, $L = 16,000$km, $g' = 2 \times 10^{-2}$m s$^{-2}$, and $\beta = 2 \times 10^{-11}$m$^{-1}$s$^{-1}$, Eq.3.7a and 3.9 give $h_w = 282$m and $H = 167$m. The near-wall speed $v_w$ is found from Eq.3.12 to be 1.26m s$^{-1}$ which is a relatively high value. Recall, however, that this is not the observed speed but rather the speed that the boundary current would have with the passages closed.

Before proceeding, a comment should be made regarding the applicability of our model to latitudes as low as 2°N. Since the Sverdrup relationship does not contain $f$, its application to 2°N is probably justified (see, e.g., HIRST and GODFREY, 1993). Similarly, since the flow is one dimensional, the geostrophic approximation is probably also valid (see, e.g., ANDERSON and MOORE, 1979; NOF and OLSON, 1993) for flows that cross the equator. However, because of the complex geography and the extreme proximity to the equator, there is some ambiguity in determining the appropriate value of $f$ with an acceptable accuracy and, in this sense, the model is a bit shaky at 2°N.

3.4.4 Numerical application of the confluence theory. The above numerical values of the depths and speeds indicate that, according to the LN and AN theories, the confluence cannot be stationary with the passages closed. Neither the near-wall depths (253m and 282m) nor the near-wall speeds (zero and 1.26m s$^{-1}$) are identical implying that the confluence must drift along the boundary. Furthermore, using LN’s migration formula one finds that, with the above numerical values, the drift speed is going to infinity. This is because there is no northward flowing retroreflecting current ($v_{lw} = 0$) that can balance the momentum flux of the southward flowing MC.

3.5 Opening the Indonesian Passages

This case is discussed in detail in NOF (1996). When the passages are open (Figs 8, 9) the condition that the boundary current transport be equal and opposite to the interior transport is no longer necessary because fluid can now leak out through the passages. Consequently, relations Eq.3.7 and 3.7a for the near-wall depths are no longer applicable. On the other hand, relation Eq.3.9 for the offshore depths outside the boundary currents is still valid because the above (equal and opposite transport) condition does not enter its derivation. This means that
Fig. 8. A diagram for the simplified geometry of the entrance to the Indonesian Seas. The Indonesian Seas are taken to be a channel running from the northeast to the southwest and crossing the equator. The dashed line shows the area that we shall later focus on (Fig. 10). (Reproduced from NOF, 1996).

Fig. 9. Schematic diagram of the separating streamlines; fluid entering the channel is shaded. Fluid north of the separating streamline corresponds to a potential vorticity $f/H_2$ whereas fluid to the south corresponds to a potential vorticity of $f/H_1$ (where $H_1 \geq H_2$).
Fig. 10. Schematic diagram of the flow in the vicinity of the Indonesian Passages. $T_1$ represents the transport of the Mindanao Current approaching the passage from the north, $T_2$ is the transport of the South Equatorial Current approaching the passage from the south, and $T_{2c}$ and $T_{1c}$ are the fractions of these currents which enter the channel. $T_{2n}$ and $T_{1n}$ are the fractions of the approaching currents that are retroflected and flow eastward. The angle $\gamma$ is the tilt of the coastline. The thick dashed line indicates the boundary of the integration area. The Halmahara Eddy (HE) and the Mindanao Eddy (ME) do not enter the computations explicitly because they are situated away from the boundary of the integration region. $y_0$ is the distance from the origin of the coordinate system (corresponding to the intersection of the coastline and the axis of the retroflected current) and the equator. It is roughly 300-400 km, whereas the average Rossby radius is 150-200 km so that the region of interest (shaded) does not contain the equator.

The offshore depths are the same as those previously derived (167 m and 253 m) so that the geostrophic transport of the NECC remains the same as in the case of the closed passages (41 Sv).

The condition of continuous Bernoulli all the way along the wall (discussed in LN) is also no longer relevant because the (previously continuous) flow is now broken into two separate branches. Because of these, the MC and the SEC can now leak out (to the Indian Ocean) any "unbalanced" portion of their transport and so adjust themselves to a state where the entire area of convergence is stationary. As mentioned, the solution for this case was found by NOF (1996) using techniques identical to those previously described (Fig. 10). We begin by noting that the matching of velocity and depth along $\hat{y} = 0$ in region 3 immediately gives us the entire
solution for this area. Namely, the general solution for region 3 is
\[ u_{3n} = A_{3n}e^{\gamma R_d} ; \quad h_{3n} = H_2 + \frac{R_d}{g} A_{3n} e^{\gamma R_d} \]
\[ u_{3s} = A_{3s}e^{\gamma R_d} ; \quad h_{3s} = H_1 - \frac{R_d}{g} A_{3s} e^{\gamma R_d} \]
where "n" and "s" denote the northern \((\gamma > 0)\) and southern \((\gamma < 0)\) regions. This system contains two unknowns, \(A_{3n}\) and \(A_{3s}\), which are the integration constants associated with the northern or southern flows and can be easily obtained by taking into account that both the depth and the speeds must match at \(\gamma = 0\). One finds,
\[ A_{3n} = A_{3s} = \frac{g'(H_1 - H_2)}{f(R_d_1 + R_d_2)} . \]

Since the flow in region 1 is specified and the flow in region 3 is now known, we can proceed and calculate the fraction of this flow which enters the channel. This is done simply by subtracting the offshore transport from the specific upstream transport. It gives,
\[ T_{1c} = \frac{g'(H_1 - H_2)}{2f} \left[ \frac{A_1}{fR_d_1} \left( 2 - \frac{A_1}{fR_d_1} \right) + \left( \frac{H_2}{H_1} - 1 \right) \right] . \] (3.13a)

The next step is to employ the integrated momentum, which takes the form
\[ \int_0^{\infty} h_1 v_1^2 dy + \int_0^{\infty} h_2 v_2^2 dy + \cos \gamma \int_0^{\infty} h_3 v_3^2 dy = 0 , \]
where the hats (\(^\wedge\)) indicate association with the tilted coordinates system. Note that the momentum flux of the flow entering the channel does not enter the integrated momentum balance because it is directed along the channel rather than along the coast. Incorporation of the known general solution into the integrated momentum balance and an application of the Bernoulli to the separating streamline which intersects the stagnation point \(S_p\) (Fig. 9) gives
\[ \frac{H_1}{2} A_1^2 - \frac{R_d}{g} A_1^3 + \left( \frac{H_2}{2} A_2^2 - \frac{R_d}{g} A_2^3 \right) \]
\[ + \left[ \frac{g'(H_1 - H_2)}{2fR_d_1 + R_d_2} \right]^2 - \frac{R_d_1 - R_d_2}{g} \left[ \frac{g'(H_1 - H_2)}{f(R_d_1 + R_d_2)} \right]^3 \cos \gamma = 0 . \] (3.13b)

This cubic equation provides the solution for \(A_2\) in terms of \(A_1\) and the other known variables and, hence, closes the problem. It turns out that, except for the trivial solution which does not sense the presence of the channel (and for separated jets that approach the wall rather than going away from it), it has only one physically relevant solution. Understanding this physically relevant root is not a trivial matter, however, as we shall shortly see. For this reason, the solution will be described in a graphical manner (Fig. 11). For given off-shore depths north and south of the channel and a given SEC transport \((T_1)\), one finds the necessary MC transport \((T_2)\), and the transport that enters the channel from the north \(T_{2c}\). The transport that enters the channel from the north \((T_{2c})\) can then be computed as a residual because the resulting retroflected \((NECC)\) transport \(g'(H_1 + 2 - H_2)/2f\) is known. With \(H_1/H_2 = 0.66, \gamma = 45^\circ\) and an average value of \(f = 0.9 \times 10^{-8}s^{-1}\) the Indonesian situation corresponds to an MC with a transport of 27Sv and a specified SEC which is carrying 26Sv. This convergence of 53Sv yields a NECC of 41Sv and a throughflow of 12Sv. From the total of 12Sv that enter the
Fig. 11. The transport approaching the Indonesian channel from the north ($T_2$) and its branch which enters the channel ($T_{2c}$) as a function of the depth ratio $H_2/H_1$ and the tilt of the coastline $\gamma$. The plot corresponds to the Indonesian Throughflow where the transport approaching the channel from the south (i.e. the SEC) is $26\text{Sv}$. For our application, $H_2/H_1 = 0.66$ and $\gamma = 45^\circ$ so that $T_2 = 27\text{Sv}$ and $T_{2c} = 11\text{Sv}$. 
channel, 11 Sv originate in the MC and one Sverdrup originates in the SEC. All of these aspects are in agreement with the observations (Gordon, 1995); however, why the system “chooses” the particular steady state described above is not entirely clear.

Before proceeding and summarizing our results, it is appropriate to comment on the validity of our assumptions. Most of the assumptions used are straightforward except perhaps the neglect of friction. It is possible to show, however, that, as a first approximation, the neglect of friction is justified for both the confluence in the western equatorial Atlantic and the confluence in the western Pacific. To do so, it is necessary to show that the computed flow-force along the wall $\int \rho u^2 dy$ is greater than the frictional force associated with the bottom boundary layer. In other words, it is necessary to show that,

$$h v^2 R_d \gg (\tau/\rho) R_d \ell,$$

where $\ell$ is the length of the integrated region (several Rossby radii) and $\tau/\rho = 0.0016 (u^*)^2$ (here, $u^*$ is the frictional velocity). The left hand side of Eq. 3.14 represents the computed flow-force whereas the right-hand side represents the integrated force associated with the stress along the bottom. Taking $u^* \approx 0.1 \text{ m s}^{-1}$, $\ell \sim 5 R_d$, $R_d \approx 200 \text{ km}$, $v \sim 0.7 \text{ m s}^{-1}$ and $h \sim 200 \text{ m}$ which are typical values for both the Atlantic and the Pacific, we find that Eq. 3.14 translates to $6 \approx 1$ as should be the case. Note that, as should be the case, the frictional velocity $u^*$ was taken to be an order of magnitude smaller than the mean flow above.

4. SUMMARY

The western equatorial Atlantic and the western equatorial Pacific are geographically very similar and, yet, are dynamically very different. Both equatorial oceans have a western boundary that is slanted in the northwest-southeast direction. Also, both oceans contain a northwestward flowing western boundary current (Fig. 1) that crosses the equator, retroreflects and turns eastward around 4°N as a result of a vanishing wind stress curl over the ocean interior (the NBC in the Atlantic and the SEC in the Pacific). Despite these fundamental similarities, the retroreflection in the Atlantic produces and sheds large retroreflection eddies every few months whereas the retroreflection in the Pacific does not usually shed rings.

To explain this fundamental difference, we have first presented a recently developed theory (Noé and Pichevin, 1996) which showed why NBC rings are produced and shed in the first place. To develop the theory we used an inviscid nonlinear layer-and-a-half model (Figs 2 and 3) integrated over one period of eddy generation (relations Eqs 2.9 and 2.10). Together with justifiable assumptions regarding the structure of the eddies exiting the generation area, we showed that the production of eddies is unavoidable. This is so because only eddies can compensate for the momentum flux of the approaching and retroreflecting NBC.

We then proceeded to the western Pacific and assumed, temporarily, that the Indonesian Passages are closed. Although this is not a realistic situation, it helps one understand the processes in question because it simplifies the problem by capturing only some of the western Pacific dynamics. It is, therefore, a useful first step. We showed that, under such conditions, the southward flowing MC (which does not have a counterpart in the Atlantic) can compensate for the northward momentum flux associated with the SEC (see, e.g., Figs 5 and 6 and relation Eq. 3.1). Under such conditions, the MC prevents the formation and shedding of rings in the western Pacific.

One can, of course, ask the more fundamental question of why there is no counterpart for the MC in the Atlantic. The answer to this question is probably related to the conveyor
operation. If one accepts the point of view that in the Pacific there is no production of deep water then one concludes that all surface water which enters the north Pacific must also exit the north Pacific as surface water. This implies that the mass flux entering the NECC from the south must later return as a western boundary current from the north and this current is the MC. In the north Atlantic, on the other hand, entering surface water does not return to the south as surface water. Instead, because of cooling, it returns as deep water, so that there is no compensatory surface boundary current (to the south) equivalent to the MC.

Although the above scenario of momentum compensating MC is reasonable, it is incomplete because most confluence zones are not “balanced” (in the sense that the momentum flux and the along-wall Bernoulli of the opposing currents are not identical) and, consequently, they drift along the coast (see relation Eq.3.5 and Fig.7). (Note that the balance condition translates to opposing currents with identical long-wall speeds and identical near-wall depths.) We have shown that not only is the confluence in the western Pacific, as expected, imbalanced but also that if the passages were closed, it would drift very rapidly toward the south. With the passages open, however, the confluence can leak the excess “unbalanced” portion of the approaching currents (Figs 8, 9, 10 and 11) and remains stationary (see, e.g., relation Eq.3.13). One concludes, therefore, that it is the Indonesian Passages that are responsible for the confluence’s relative stationarity and its position.

It has also been shown that, for the Indonesian Throughflow, the amount of fluid originating in the north must be an order of magnitude greater than that originating in the south. Specifically, for the numerical values typical for the throughflow, about 1Sv originates in the south and 11Sv in the north. The proposed model does not preclude the existence of steady eddies (such as the Halmahara and Mindanao) next to the entrance. In addition, it is suggested that it is no accident that the retroflection is situated immediately to the east of the Indonesian Passages. This is a consequence of the imbalance in the flow-force exerted by the two opposing jets which can be avoided only by leaking the unbalanced portions into the Indian Ocean.

Finally, two additional points should be made with regard to the above statements. First, it is not said here that there are no retroflection eddies in the Pacific because of the Indonesian Throughflow. There would not have been any retroreflection eddies even if the Indonesian Passages had been closed. Under such conditions, however, the confluence would not have been stationary. Second, one needs to keep in mind that our models are simplified so that they do not capture all the processes active in the ocean. For instance, it is quite possible that, in reality, what keeps the confluence stationary is not the leakage through the passages but rather the various islands (associated with the local complicated geography) that lock the flow in one place.

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6. REFERENCES


### APPENDIX

**LIST OF SYMBOLS**

- \( A_{ss} \): Integration constants associated with the northern or southern flows.
- \( a \): Eddy width along the \( \hat{x} \) axis.
- \( b_1, b_2 \): Distances between the eddy edges and the coast (Fig.3).
- \( C \): Eddy migration rate.
C*  nondimensional eddy migration rate
D  nondimensional near-wall depth
F  coefficient of interfacial friction
f  Coriolis parameter \((f_0 + By)\)
g'  "reduced gravity", \((\delta \rho / \rho)g\)
H  upstream undisturbed depth
\(H_e\)  eddy depth
\(H^p\)  potential vorticity depth of both the current and the eddies
\(h\)  thickness of light water
\(h_e\)  eddy depth
\(h_w\)  near-wall depth
\(H_l\)  offshore undisturbed depth of the poleward retroflecting current
\(H_c\)  offshore undisturbed depth of the counter current
\(H_{lw}\)  near-wall undisturbed depth of the poleward retroflecting current
\(H_{cw}\)  near-wall undisturbed depth of the counter current
\(i\)  length of the integrated region
L  combined width of approaching and retroflecting current
L_p  Pacific basin's length
\(M_{1,2}\)  momentum flux of the approaching current and the counter current, respectively
Q  approaching current transport
r, \(\theta\)  polar coordinates
R  eddy diameter
\(R^*\)  nondimensional eddy diameter
\(R_d\)  Rossby deformation radius of the parent current
\(R_{de}\)  Rossby deformation radius of the eddy
S  integration area
T  period of eddy generation
\(T^*\)  nondimensional periodicity
\(T_{1,2}\)  volume transports of the main current, counter current, and separated current, respectively
U,V  vertically integrated transports in the x and y directions
\(u,v\)  velocities in Cartesian coordinates
\(u^*\)  frictional velocity in the bottom boundary layer
\(v_w\)  initial near-wall speed
\(\gamma\)  orbital velocity
\(\chi, \hat{\chi}\)  axes of tilted coordinates system pointing eastward and northward
\(\alpha\)  unknown of order unity measuring the size of the resulting eddies
\(\varepsilon\)  small parameter equal to \(\beta R / f_0 \ll 1\)
\(\phi\)  boundary of integration area S
\(\gamma\)  tilt of the coastline (see Fig.10)
\(\rho\)  upper layer density
\((\rho + \Delta \rho)\)  lower layer density
\(\psi\)  steady streamfunction (defined by \(\partial \psi / \partial y = -uh; \partial \psi / \partial x = vh\))
\(\psi^\star\)  time-integrated streamfunction
\(\tau_x\)  x component of wind stress along ocean surface
\(\xi, \eta\)  relative vorticity of the current
\(\zeta_e\)  relative vorticity of the eddy
\(\phi\)  counterclockwise integration