Fast wind-induced migration of Leddies in the South China Sea

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Abstract

Eddies off the Strait of Luzon (which we term here “Leddies” in analogy to Teddies originating from the Indonesian Throughflow) are formed rapidly and migrate swiftly. Their migratory rate (~ 10-20 cm/s) is an order of magnitude faster than that of most eddies of the same scale (~ 1 cm/s). On the basis of observations, it has been suggested earlier that the rapid generation process is due to the Southeast Monsoon.

Here, we place this earlier suggestion on a more solid ground by developing both analytical and process oriented numerical models. Because the eddies are formed by the injection of foreign, lighter Kuroshio water into the South China Sea (SCS), we model them as lenses, i.e., “bullets” that completely en-capsule the mass anomaly associated with them. It turns out that the rings migrate at an angle $\alpha$ (between zero and 90°) to the right of the wind direction [i.e., $\tan^{-1}(2 - \gamma)f^2R/8g'C_D$, where, in the conventional notation, $\gamma$ is the vorticity, $R$ the eddy radius and $C_D$ the interfacial friction coefficient along the lens’ lower interface]. Their fast migration speed is given by $2(\tau_s/\rho_w)(\sin \alpha)/fH$, where $\tau_s$ is the wind stress on the surface, $\rho_w$ the water density and $H$ is the maximum eddy depth. With high interfacial drag (i.e., large $C_D$) the rings move relatively slowly (but still a lot faster than Rossby waves) in the wind direction, whereas with low drag they move fast at 90° to the right. These analytically predicted values are in good agreement with our isopynic numerical simulations.
1. Introduction

The Luzon Strait is a rich source of eddies originating from the Kuroshio (Fig. 1). In analogy to eddies originating from the Mediterranean outflow (Meddies), the Red-Sea (Reddies), and those from the Indonesian Throughflow (Teddies), we term these eddies “Leddies”. Farris and Wimbush (1996) were the first to point out (on the basis of SST analysis) that the rings production occurs primarily during the winter. They were also the first to attribute this fast generation to the strong Northeast Monsoon. This was later verified by the altimetry analysis of Wu and Chiang (2007) and Liang et al., (2008), who argued that, during the winter, there are five times more rings than during the summer. Using in-situ hydrographic and buoy data, Wang et al., (2008) determined that, during its relatively short ~ 40 days life span, the Leddies migrate westward at a mean speed of about 10 cm/s but the instantaneous migration speed could reach as much as 20 cm/s\(^{-1}\) or could be as low as 6 cm/s. It is mentioned here in passing that the Luzon Strait is very different from other outflows that produce eddies (see e.g., Durland et al. 2009) primarily because it is dominated by strong wind.

Although some authors (understandably) claim that Leddies migrate at the Rossby wave speed (e.g., Wang et al., 2008), we argue here that this is not really the case and suggest that, instead, the strong wind induces the observed speeds. Because of the rings’ proximity to the equator, they are very shallow (~ 60 m, see Fig. 6 of Wu and Chiang, 2007, and Fig. 8 in Wang et al., 2008) and have a density anomaly of three parts per thousand (see e.g., Qu et al., 2006). With a Coriolis parameter of \(0.5 \times 10^{-4} \, s^{-1}\) (relevant to the Luzon Strait which is situated at 21°N), the Rossby radius \(R_d\) associated with the
Leddies is about 27 km. (This needs to be distinguished from the general Rossby radius in the SCS, which is based on a deeper general thermocline depth, giving a somewhat larger Rossby radius.)

A Rossby wave speed \( \left( \beta R_d^2 \right) \) of less than 1.5 cm/s is associated with the above Leddies’ Rossby radius and this is an order of magnitude smaller than the observed 10-20 cm/s mentioned above. In fact, the discrepancy between the two is even worse because the mean thickness of any anticyclone is smaller than the maximum thickness at the center implying a migration rate smaller than the Rossby wave. (Nof, 1981, argues that, for most anticyclones of that nature, it is roughly half the Rossby wave speed, giving a Leddy migration speed of less than 1 cm/s.)

The high variability of the migration rate, 6-20 cm/s during the relatively short 40 days lifetime (Wang et al. 2008), also places the Rossby wave idea on a shaky ground. The SCS stratification cannot possibly change so rapidly suggesting again that it must be the wind, which has a much shorter time scale, that drives the eddies westward. In what follows we will present an analytical solution for the wind-induced ring migration rate (Sections 2, 3 and 4) as well as numerical simulations (Section 5) and show that both support the wind-driven Leddies idea. The results are summarized and discussed in Section 6.
2. Formulation

Consider an inviscid lens (on an \( f \)-plane) whose dimensions and strength in the absence of wind and \( \beta \) are known. This no-wind and no-\( \beta \) inviscid state is circular, steady and stationary (\( C = 0 \)). Both the orbital speed and thickness (\( h \)) depend on the potential vorticity and both are known. The ring has zero thickness along its rim and is floating on top of an infinitely deep resting fluid. It is then subject to a uniform surface wind stress \( \tau \), and is migrating in response to this wind action. It is assumed that it does so steadily at an unknown speed \( C \) forming an unknown angle \( \alpha \) with the wind direction (Fig. 2). Given that the ring’s time scale is \( \sim o (f^{-1}) \), we expect the ring to adjust quickly to the wind.

Much can be learned at this point by vertically integrating individual terms in the equations of motion (across the lens) in a coordinate system moving steadily with the ring at speed \( C \). Recall that, in a moving coordinate system, there are two Coriolis terms, one involves the migration speed \( C \), and the other is associated with motions relative to the moving lens, e.g.,

\[
\begin{align*}
    u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu + fC &= -g' \frac{\partial h}{\partial y} + \nu \frac{\partial^2 v}{\partial z^2},
\end{align*}
\]

where, in the conventional notation, \( u \) and \( v \) are the horizontal speeds, \( g' \) the “reduced gravity”, \( g \Delta \rho_w / \rho_w \) (with \( \Delta \rho_w \) being the density difference and \( \rho_w \) the density), and \( \nu \) is the viscosity.
First, note that a vertical integration of the Coriolis term that does not involve $C$ from the lens lower to upper boundary gives zero because the lens is an enclosed capsule (and steady) so that any Ekman flux near the surface must be compensated for by a returning flow below. This situation needs to be distinguished from that in a small-amplitude (quasi-geostrophic) eddies where there is a free exchange of mass between the eddy and the environmental fluid so that a surface Ekman layer is not necessarily compensated for by a return flow underneath (see e.g., Dewar and Flierl, 1987). It also needs to be distinguished from the integration of the Coriolis term involving $C$, which does not vanish but rather gives $fCh$.

Second, integration of the vertical frictional term ($\nu u_z$ or $\nu v_z$) across the lens results in two terms, the wind-induced stress on the surface and a compensating interfacial stress on the lower interface. The latter results from the fact that the lens is migrating whereas the fluid below is stagnant. Internal horizontal friction within the lens itself does not enter the above relationship. Also, note that, since the fluid around and below the lens is infinitely deep, the wind is only affecting the lens and not the lower fluid.

In this scenario, there are three forces acting on the ring--the integrated wind stress acting on the lens upper surface pushing it in one direction, the integrated Coriolis force acting to the right of the direction of migration, and the interfacial drag acting on the lower interface in the direction opposite to the migration (Fig. 2). It is assumed that overall the wind imposes a small perturbation on the ring, i.e., the resulting migration speed $C$ is small compared to the orbital speed and the distortions from a pure circle are also small.
3. Equations

We first note that a common expression for the frictional interfacial drag (that opposes the motion) is,

\[ \iint C_D \vec{U} \cdot d\vec{A} = \iint C_D (u + C)(v^2 + (u + C)^2)^{1/2} d\sigma, \]

where the integration is done over the area of the Leddy, \( \vec{U} \) is the velocity vector, and \( u \) and \( v \) are the speeds in the \( x \) and \( y \) directions in the (approximately steadily) moving coordinate system. The coordinate system is moving with the ring at speed \( C \). We expand now the term inside the integral and neglect terms of \( O(C^2) \) compared to terms of \( (uC) \),

\[
(u + C)(v^2 + u^2 + 2uC)^{1/2} = (u + C)(u^2 + v^2)^{1/2}[1 + 2uC / (u^2 + v^2)]^{1/2} \\
= (u + C)(u^2 + v^2)^{1/2}[1 + uC / (u^2 + v^2)] = u(u^2 + v^2)^{1/2} + C(u^2 + v^2)^{1/2} + u^2 C / (u^2 + v^2)^{1/2} \\
= u(u^2 + v^2)^{1/2} + C(2u^2 + v^2) / (u^2 + v^2)^{1/2}. \quad (2)
\]

It is important to realize at this point that the surface integral of the first term in the last line of (2) vanishes because \( u \) is asymmetrical so only the second term is left in (1). This vanishing reflects the condition that the drag associated with the orbital speed of the lens does not enter the migration calculation on its own.

Under such conditions, there are two equations for the integrated forces acting on the ring. The first involves the balance in the direction of migration (wind stress component
balancing interfacial drag) and the second is the balance in the direction perpendicular to the migration (wind stress component balancing Coriolis):

\[
\frac{(\tau_s / \rho_w)}{\cos \alpha} \int r dr d\theta = C_D \int \int (2u^2 + v^2)(u^2 + v^2)^{-1/2} Cr dr d\theta \tag{3}
\]

\[
\frac{(\tau_s / \rho_w)}{\sin \alpha} \int r dr d\theta = f C \int \int hr dr d\theta , \tag{4}
\]

where \( h \) is the ring’s thickness, \( C_D \) the interfacial drag coefficient, and the surface integration (in polar coordinates) is done from zero to \( R \) (the ring radius) and zero to \( 2\pi \). These two equations give the solution for the two unknowns, \( C \) and \( \alpha \). The reader who wondered about the pressure term is reminded that there is no net pressure force acting on the lens because the thickness vanishes around the rim. Namely, using Green’s theorem, the integration of the pressure term over the lens yields,

\[
\oint \frac{\partial}{\partial y} \left( \frac{h^2}{2} \right) dxdy = -\frac{1}{2} \oint h^2 dx = 0 ,
\]

where the integration along the boundary (RHS) is done in a counterclockwise manner (though this does not really matter because \( h = 0 \) along the boundary).

4. **Solution**

Dividing (4) by (3) gives,
\[ Tan \alpha = f \int \int hr dr d\theta / C_p \int \int (2u^2 + v^2)(u^2 + v^2)^{-1/2} rdr d\theta \]  

(5),

which can be used to calculate the angle because the eddy’s structure is known in advance (from the no-wind case). We observe that the direction of migration is independent of the wind stress \( \tau_s \).

Taking the simple case where the orbital speed \( V_o \) is linear,

\[ V_o = -\gamma fr / 2, \quad \gamma \leq 1 \]  

(6),

where \( \gamma \) is the vorticity, and using,

\[ \frac{V_o^2}{r} + fV_o = g' \frac{dh}{dr}, \]

we find that the ring’s volume, and radius are,

\[ V = \gamma(2 - \gamma)\pi f^2 R^4 / 16g'; \quad R = 2\sqrt{2}R_d / \gamma^{1/2}(2 - \gamma)^{1/2}; \quad R_d = g'H / f, \]  

(7)

where \( H \) is the ring’s maximum thickness at the center. For \( \gamma = 1 \), the system corresponds to a zero potential vorticity eddy and, in the more general case, the potential vorticity is not uniform. In the zero potential vorticity case,
\[ V = 4\pi f^2 R_a^2 / g'; \quad h = H - f^2 r^2 / 8g'. \]  

(8)

The denominator in (5) gives: \( \pi\gamma C_D f R^3 / 2 \) whereas the numerator in (5) is just \( f \) times the lens volume so (5) ultimately gives our desired expression for \( \alpha \),

\[ \tan \alpha = (2 - \gamma) f^2 R / 8g' C_D. \]  

(9)

Several limits should now be considered. First, small rings (small \( R \)) will move at the direction of the wind whereas large rings will move at 90 degrees to the right. Most rings will be in between those two limits. Highly frictional interface (large \( C_D \)) also corresponds to rings moving at the direction of the wind and weakly frictional interface corresponds to rings moving at 90 degrees to the right. Along the equator \( (f = 0) \) rings migrate at the direction of the wind.

For a 120 km radius ring, a Coriolis parameter of \( 0.5 \times 10^{-4} s^{-1} \) (relevant to the Luzon Strait), \( \gamma = 0.2 \) (corresponding to a vorticity of \( 0.1f \)), and \( C_D = 0.002 \), the tilt relative to the wind direction will be roughly 48 degrees to the right. Here, the Rossby radius is about 27 km and, due to the low latitude, the central depth \( (H) \) is only 60 meters. Once \( \alpha \) is known, the speed can be calculated from either (3) or (4). Noting that the ratio between the two integrals in (4) is \( H/2 \) regardless of the vorticity \( \gamma \), and using \( \tau_s = \rho_A C_D U_{10}^2 \), where the subscript “A” corresponds to “air”, we find from (4),
\[ C = 2 \left( \frac{\rho_A}{\rho_w} \right) C_{DA} U_{10}^2 \left( \sin \alpha \right) / fH , \] (10)

which is valid for all \( \gamma \). Since \( \alpha \) depends on \( \gamma \), (10) is not truly independent of \( \gamma \). For the parameters mentioned above, \( U_{10} \) of 15 m/s (strong monsoon wind) and a wind/water drag coefficient of again 0.002, \( C \) is about 22 cm/sec, which is very reasonable. Here, the maximum orbital speed is 60 cm/s, which, as required, is larger than the migration rate.

To summarize this subsection, it can be seen from (10) that the effects of winds on lenses can only be important when the rings are shallow (small \( H \)), the latitude is small (small \( f \)) and the winds are strong (high \( U_{10} \)). With high interfacial drag the rings move slowly in the wind direction whereas with low drag they move fast at 90\(^\circ\) to the right. This situation seems to hold in the Luzon Strait but will not hold for Gulf Stream rings where the effect will most likely be three orders of magnitude smaller because the lenses involve a very large amount of fluid so \( H \) is much larger. Also, \( f \) is larger and \( U_{10} \) is much smaller. We shall return to this important point later.

5. **Numerical simulations**

The numerical model is the shallow water, isopycnic coordinate general circulation model of Bleck and Boudra (1986). The advantage of this coordinate system is that lateral diffusion is along isopycnal surfaces, where mixing of material properties by eddies in the stably stratified parts of the oceans mostly occurs (Chassignet et al, 1990).
The suitability of this model for ocean eddies studies was examined by Chassignet et al. (1990), and Chassignet and Cushman-Roisin (1991).

The model consist of a momentum equation and a continuity equation:

$$\frac{\partial \vec{V}}{\partial t} + \frac{\nabla \rho \vec{V}^2}{2} + (\zeta + f) \vec{k} \times \vec{V} = -\nabla \rho M + \delta \frac{\partial \tau}{\partial z} + A_M h^{-1} \nabla \rho \cdot (h \nabla \rho \vec{V})$$

(11)

$$\frac{\partial h}{\partial t} + \nabla \rho \cdot (h \vec{V}) = 0,$$

(12)

where $M = g \zeta + \rho \delta$ is the Montgomery potential, $h$ is the thickness of a layer of constant density; $\delta$ is the specific volume ($\rho^{-1}$), $\tau$ is the stress, $A_M$ is lateral viscosity, and the subscript $\rho$ indicates derivatives on surfaces of constant density. For more details, the reader is referred to (Bleck and Boudra, 1986).

The model is configured in a two-layer 2000 km $\times$ 2000 km square domain on an $f$ plane with a grid spacing of 10 km. It is initialized with a Gaussian distribution of the upper layer thickness, $h = H e^{-r^2/2L^2}$, where $r$ is the radius (measured from the lens’ center), $L$ is the radius of maximum velocity, and $H$ is the interface thickness at the lens center. The upper layer thickness $h$ is zero at the edge of the lens. The ring radius and $H$ are taken as 140 km and 100 m, resembling the anticyclonic eddies separated from the Kuroshio in the Luzon Strait.
The reduced gravity $g'$ is taken to be (on the high side) 0.0294 ms\(^{-2}\) corresponding to density variation of three parts per thousand [see e.g., Qu et al., 2006, (their Fig. 7a) showing water density changes from 22 to 27 in the upper 1000 m]. The initial second (lower) layer thickness is 5000 m, which is 50 times larger than the maximum thickness of the lens. The model is forced with zonal wind of 0.5 N/m\(^2\), which is the typical value of winter monsoon in the SCS, corresponding to a wind speed of 15 m/s at 10 meters height (see e.g., Liu and Xie, 1999). The Coriolis parameter was taken to be $0.5 \times 10^{-4}$ s\(^{-1}\).

In addition to the small drag that the model contains for stability purposes, a quadratic drag is applied to the interface between the two layers. To avoid a significant reduction of the orbital speed by this additional drag, it was applied only to the migration speed of the lens but not the orbital speed [i.e., in accordance with the analysis presented in the beginning of Section 3, the first term in the last line of (2) was ignored].

Three numerical experiments, differing in the value of the drag coefficient $C_D$, were executed. Since each individual experiment includes numerous data points, we feel that this number of experiments is adequate. The quadratic drag coefficient $C_D$ is zero in experiment A, 0.001 in experiment B and 0.002 in experiment C. The numerical results are shown in Fig. 3, 4, 5, 6, 7 and 8. Fig. 3 displays the decay of the central thickness, which comes about through the flattening of the eddy. This flattening is both due to the numerical friction, which, as mentioned, is added for stability, as well as the physical interfacial drag $C_D \overline{\|\vec{U}\|}$, which is generated in response to the strong wind. It is similar to the analytics and, presumably, the ocean.
**Fig. 3** shows that, under these conditions, the central thickness decreases with time due to the two frictional terms but the reduction is not very significant (< 30%). **Fig. 4** illustrates the migration path. It vividly shows that, as the analytics predicts, increasing the friction (interfacial drag) slows the rings down and decreases their deflection to the right. Conversely, decreasing the interfacial friction speeds the migration and increases the deflection. **Fig. 5, 6, and 7** display a comparison of the numerical and analytical migration angle (right panels) and the migration speeds (left panels). The agreement of the angles is excellent (deviations are less than ~ 5%) and the agreement of the speeds is good (deviations are less than ~ 30%).

As should be the case, the numerical speed is always smaller than the analytical (due to the numerical friction). **Fig. 8** shows the thickness contours as a function of time for our largest friction experiment. In a sense, this is our “worst” experiment. It illustrates that the ring breaks down and splits into two rings (one large and one small) on day 20. Despite this split, the agreement of the analytics and numerics is still reasonable.

### 6. Summary and Discussion

In contrast to most rings and eddies in the world ocean which do not display a strong seasonal variability, the Luzon Strait eddies (**Fig. 1**) exhibit a pronounced variability with much more eddies being formed during the winter than during the summer. Since the stratification is usually *smaller* in the winter, it is unlikely that this enhanced rings
generation is due to increased instability. Instead, as pointed out first by the observational analysis of Farries and Wimbush (1996), and later by the regional numerical work of Metzger and Hurlburt (1996) and Zhao et al., (2009), it is probably due to the strong Northeast Monsoon, which blows toward the southwest during the winter, forcing the Kuroshio water westward into the SCS.

A rough calculation indicates that the Northeast Monsoon winds are so strong that (through their Ekman flux) they can produce up to two Leddies in three months. Here, we use an analytical approach as well as process-oriented numerical models to show that these winds induce a very fast eddy migration rate enabling the newly formed Leddies to quickly escape from the generation area (near the strait) thus freeing the region for the formation of the next ring in line. This process enhances the production of rings because it allows them to form much faster.

Although some authors suspected that Leddies migrate at the Rossby wave speed, we argue here that this is not really the case and suggest that, instead, the strong wind induces the observed speeds. We note first that a Rossby wave speed ($\beta R_d^2$) of less than 1.5 cm/s is associated with the Leddies’ Rossby radius and this is an order of magnitude smaller than the observed 10-20 cm/s mentioned above. Second, we note that the high variability of the migration rate, 6-20 cm/s during the relatively short 40 days lifetime (Wang et al. 2008), also places the Rossby wave idea on a questionable ground. The SCS stratification cannot possibly change so rapidly suggesting again that it must be the wind, which has a much shorter time scale, that drives the eddies westward.
We chose to look at a lens rather than the familiar small-amplitude quasi-geostrophic eddy because Leddies are associated with the injection of foreign warm and fresh water into the SCS. From a topological point of view, such injection implies that only eddies which completely en-capsule the anomalous water can exist in the SCS. Namely, we consider eddies containing warm water whose thickness is finite at the center but zero along the rim. These eddies are embedded in a very deep layer of slightly heavier water (Fig. 2) and are inherently nonlinear because \((v_0 / fr) \sim o(1)\) and the amplitude variation is of order unity.

Using the balance of forces in the direction of migration as well as the direction perpendicular to it and assuming that the wind stress acting on the Leddies is compensated for by interfacial stress along the ring lower boundary, we derived simple relationship for the migration speed as well as the migration angle relative to the wind direction (relations 9, 10). They show that, with high interfacial friction, the Leddies move relatively slowly (but still fast relative to a Rossby wave) in the direction of the wind whereas with low interfacial friction they move very fast at 90° to the right of the wind (looking downwind). They also show that small rings move faster than larger rings because they are lighter and, hence, are more susceptible to the wind.

One can easily see from our solution that most rings in the world ocean (e.g., Gulf Steam Rings, Agulhas rings) will not be subject to a very significant wind effect because the wind is in general relatively small and the rings are usually fairly large. Specifically, with
$H \sim 500 - 1000 m$ and wind at 10 meters of 3-5 m/s, the rings will migrate at a speed of the order of 1 mm/s. In that sense, Leddies are an anomaly because they are shallow (< 100 m), they are in low latitude ($\sim 20^\circ N$) and they are subject to strong Monsoon winds ($U_{10} \sim 10 - 20 m/s$). As a result, their wind-induced migration is much larger, roughly, 10-20 cm/s. For a 120 km radius ring, a Coriolis parameter of $0.5 \times 10^{-4} s^{-1}$, $\gamma = 0.2$ (corresponding to a vorticity of 0.1f), and $C_D = 0.002$, the tilt relative to the wind direction will be roughly 48 degrees to the right. Given that the Northeast Monsoon is blowing toward the southwest, this 48 degrees tilt implies that the Leddies migrate to the west, just like Rossby waves do but for a different reason. Because of the topography and the orientation of the boundary in the SCS (Fig. 1), the Leddies cannot maintain this westward migration for very long. Fairly quickly they adjust to the topography and start migrating toward the southwest.

We then proceeded with a series of process-oriented numerical experiments in which there were two kinds of frictional terms. The familiar, very small (and hopefully negligible) numerical friction that is included merely for numerical stability, and a physical interfacial friction, $C_D \overline{|U|}$, that is similar to our analytics and (hopefully) nature because the migration speeds are very high. As mentioned, to avoid unnecessary spin down in the numerics, we eliminated the first term in the last line of (2) from the numerical model. This term vanishes in the analytics due to symmetry and does not enter the balances associated with the migration.
Our numerical results are shown in Figs. 3-8. All show excellent agreement between the predicted analytical and numerical direction of migration (differences of less than 5%) and reasonable agreement of the migration speeds (discrepancies of < 30%). As should be the case, the numerical speeds are all smaller than the analytical due to numerical friction that slows the rings down. In general, the small interfacial friction experiments (i.e., small $C_D$) are in better agreement with the analytics probably because the rings stay more coherent during the runs. Our worst agreement, which is shown in Fig. 8, involves splitting of the ring into a large and small ring at day 20. Despite that split, the agreement is still reasonable.

Finally, it should be recalled that, while the analytical/numerical comparison is useful, it does not give the final answer to the Leddies drift question. Both the numerical experiments and the analytical models have their drawbacks and strengths. On the one hand, the numerical models incorporate a wider range of processes but, on the other hand, simple analytical models are more amenable to analysis and are valuable for their illumination of any changes that take place in response to other variations. The ultimate answer to the question how rings move due to strong wind action can only be provided by a careful analysis of both satellite and in-situ observations. Given the maturing stage of Quickscat and other techniques, it is hoped that both methods will be available soon.
Appendix

Symbols

$A_M$ Lateral viscosity

$C$ Migration speed of the eddy’s center

$C_{DA}$ Coefficient of interfacial drag in the atmosphere

$C_D$ Coefficient of interfacial drag

$f$ Coriolis parameter

$g'$ Reduced gravity

$H$ Eddy’s maximum thickness

$h$ Thickness of the eddy

$L$ Radius of maximum velocity in the eddy

$r, \theta$ Polar coordinates

$R$ Radius of the eddy

$R_d$ Rossby Radius

$u, v$ Horizontal velocity components

$U_{10}$ Wind speed at 10m above ocean surface

$V_\theta$ Orbital speed of the eddy

$\alpha$ Migration angle of the eddy’s center

$\rho_A$ Density of the atmosphere

$\rho_w, \Delta \rho_w$ Density of the ocean and density difference between the layers

$\tau_s$ Surface wind stress

$\delta$ Specific volume
\[ \gamma \] Vorticity of the eddy

\[ \nu \] viscosity
References:


**Fig. 1:** Schematic diagram of a Leddy formed of the Luzon Strait (adapted from Fig. 6 in Wang et al., 2008). Isobaths are for 300 m and 2000 m; the large arrow shows the direction of the northeast monsoon.
**Fig. 2a:** Schematics tope view of the eddy wind-induced migration.

**Fig. 2b.** A schematic cross section of a Leddy in the direction of migration. The shown wind stress is the stress component in the drift direction ($\tau \cos \alpha$). It is balanced by interfacial stress on the bottom of the lens.
Fig. 3. Maximum eddy thickness (in meters) as a function of time for the three numerical experiments, A (solid line, $C_D = 0$), B (dashed line, $C_D = 0.001$), and C (dotted line, $C_D = 0.002$).

Fig. 4. Ring numerical trajectories calculated from the center of mass (every 5 day from day 1 to day 60). Stars, circles and triangles mark the trajectories of experiment A, B and C. Large arrow marks the direction of the wind. Note that high-drag rings (C) are more aligned with the wind direction than the low-drag rings (A) but, as expected, they migrate more slowly.
Fig. 5. Left panel: Analytical (solid line) and numerical migration speed (dashed line) of Experiment A. Right panel: Angle of migration ($\alpha$) of experiment A.

Fig. 6. The same as for Fig. 5 except that it is for experiment B.
Fig. 7. The same as for Fig. 5 except that it is for experiment C.
Fig. 8. Thicknesses (in meters with 10m interval) and velocity (in m/s) of experiment C (from day zero to day 30, in 10 days increments). The contour interval is 10 m. Note that this large friction experiment is the “worst” of our three experiments in the sense that it displays the largest variability.