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On the ellipticity of isolated anticyclonic eddies

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ABSTRACT

A simplified two-layer model is considered in order to describe the behavior of anticyclonic lens-like eddies embedded in a horizontally sheared flow. Attention is focused on light eddies (such as warm-core gulf stream rings) embedded in an infinitely deep fluid whose speed varies linearly in the horizontal direction. Non-linear solutions are constructed analytically using a power series expansion.

It is found that a lens-like eddy embedded in a horizontally sheared flow, on an f plane, migrates at the average speed of the mean flow. As a result of the shear, the ring's shape is an ellipsoid whose orientation depends on the environmental flow. When the environmental shear is cyclonic, the ellipse is oriented perpendicular to the mean flow, and when the environmental shear is anticyclonic, the ellipse orientation is parallel to the mean flow. Although the ring migrates at a speed which differs from the speed of most of the surrounding fluid, the elliptical shape is fixed so that the whole eddy does not rotate due to the shear.

A possible application of this theory to the warm-core rings found north of the gulf stream is discussed.

1. Introduction

In the north Atlantic Ocean, warm-core rings are formed by anticyclonic gulf stream meanders which close upon themselves and, subsequently, pinch off. After their formation, the rings drift westward in the slope water until they ultimately reach the continental boundary and rejoin the stream.

With orbital velocities exceeding 1 m s^{-1}, a depth of ~600 m and a length scale of ~50 km (see Fig. 1), they represent a substantial source of kinetic energy. One of the most interesting aspects of these rings is their various shapes and structures. At times, their horizontal shape is very close to a circle, but often their shape resembles an ellipse (Brown et al., 1983). In this paper we shall examine a mechanism which may lead to such an elliptical shape. Using a two-layer analytical model, we shall show that such a shape may simply be a result of the flow in which the rings are embedded. Specifically, it will be suggested that the observed elliptical shape may be caused by the horizontal shear of the environmental fluid.

Our aim in this paper is not to develop a model for a particular set of observations, but rather to develop a general model which will describe the general influence of a horizontally sheared flow on the structure and shape of lens-like rings. To do this, we shall simplify the problem to that of a warm lens-like anticyclonic eddy embedded in an infinitely deep lower layer with a horizontal shear (Fig. 2). The frequently used quasi-geostrophic equations are inappropriate for such a model because they require the interface displacement to be small, whereas the ring's displacements are of order unity. For this reason, we shall use the full non-linear equations which allow the interface to surface at a finite distance from the center.

During the last two decades, there have been a number of studies of isolated eddies and gulf stream rings. Among these studies are those of Warren (1967), Stern (1975), Firing and Beardsley (1976), Larichev and Reznik (1976), Flierl (1977, 1979), Mied and Lindemann (1979), McWilliams and Flierl (1979), Flierl et al. (1980), Nof (1981, 1982), Killworth (1983), Flierl et al. (1983) and Flierl

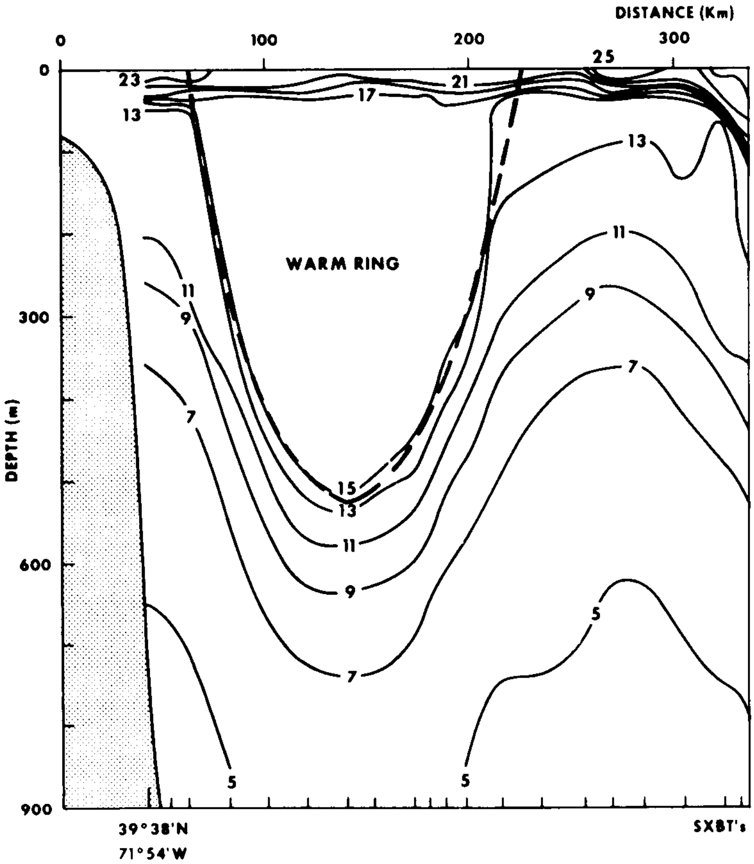

Fig. 1. The vertical structure of a warm-core ring observed north of the Gulf Stream on 29 June 1975 (adapted from Cheney (1976)). Isotherms are denoted by solid lines and the chosen interface depth (corresponding to the basic state) is denoted by the dashed line. The interface corresponds to a radially symmetric eddy with a linear swirl velocity. Following Csanady (1979) we chose the following numerical values: $g' \approx 10^{-2}$ m^{-2}; $f \approx 10^{-4}$ s^{-1}; $\hat{h} \approx 500$ m; $r_0 \approx 80$ km and $V|_{r_0} \approx 1.5$ m^{-1} (here, r_0 is the radius of the eddy and $V|_{r_0}$ is the swirl velocity along the edge).

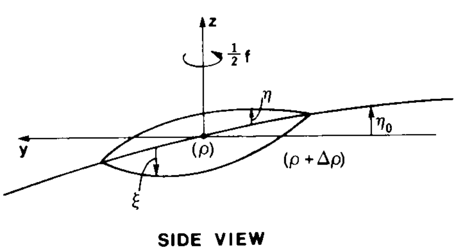

Fig. 2. A schematic diagram of the model under study. The ring is embedded in an infinitely deep lower layer with a linear horizontal shear. Consequently, it is advected and its shape adjusts to the surrounding pressure field. The free surface displacements η and η_0 are associated with the ring and environmental movements, respectively, and are measured upward. The ring's interface displacement ξ is measured downward.

(1984). In addition to these studies, which address various processes associated with isolated eddies, there have been a few attempts to determine the influence of shear on isolated vortices. Shen (1981) has looked at the general influence of horizontal shear on the dynamical balance of isolated eddies. However, he focused solely on the general dynamical balances and did not specifically address the question of the eddy's shape. In addition, his results for stratified flows are limited to quasi-geostrophic motions so that they are inapplicable to lens-like eddies. Swenson (1982) has examined the behaviour of barotropic modons and isolated vortices in the presence of horizontal shear. In contrast to Shen's analysis which does not address the eddy's shape, Swenson's study involves detailed solutions which include the shape distortions due to the shear. For isolated vortices, he considered special vorticity fields both inside and outside the vortex. As a result of this choice, the entire field is affected by the vortex. Moore and Saffman (1972) have also studied the behaviour of isolated vortices in the presence of shear. They focused on barotrophic vortices embedded in a *non-rotating* flow and found that the shape distortion depends on the sense of the environmental shear as well as the circulation within the vortex.

While all these studies are informative, they do not deal with the problem addressed in this study where a non-linear lens-like eddy is embedded in a deep sheared flow. That is to say, these studies could be applied to various vortices with a uniform depth embedded in a fluid with identical depth, but they cannot be applied to warm-core rings because their structure is baroclinic and corresponds to a lens (Fig. 1).

To model the migration and structure of the ring, we shall consider an infinitely deep mean field whose velocity is independent of depth but varies linearly in the horizontal direction. The mean field affects the eddy in two ways; first the eddy is advected at a speed which is to be determined, and secondly its structure is influenced by the shear. To compute the eddy's structure and translation, the governing equations are considered in a coordinate system moving with the eddy and their structure is simplified using a perturbation scheme in $\varepsilon = A/B$, the ratio between the environmental shear (A) and the shear that the eddy would have in the absence of a mean flow (B). These simplified equations are then solved analytically and give the desired eddy's shape as well as the migration speed. Note that the integrated equations approach used by Nof (1981, 1982, 1983a, b) is insufficient for solving the problem at hand because it focuses on the translation speed and does not give any information about the eddy's shape.

For simplicity, it will be assumed that in the absence of an environmental flow, the ring's orbital velocity corresponds to a linear distribution. Using this velocity profile and the perturbed equations, the eddy's migration speed and structure are expressed in terms of the outer field mean flow (U_0), its shear (A) and the basic eddy's shear (B). The results of the mathematical model are then qualitatively compared with the observations of Brown et al. (1983). Although there are only rough estimates of the environmental shear, the comparison reveals that the actual observed shape may result from the ring's surroundings.

This paper is organized as follows: the formulation of the problem is presented in Section 2 and the perturbation analysis in Section 3. Section 4 contains an analysis of the results and the applicability of our model to gulf stream rings. Section 5 summarizes this work.

2. Formulation

As an idealized formulation of the problem, consider the two-layer model shown in Fig. 2. The upper lens-like layer, whose density is ρ, represents the ring. We shall consider a coordinate system moving with the eddy itself at speed C; the origin of the coordinate system is located at the center of the eddy. The x and y axes are directed along and perpendicular to the mean flow, and the system rotates uniformly with angular velocity $\frac{1}{2}f$ about the vertical axis. It is assumed, and later verified, that the translation is steady and that the eddy's shape does not change in time so that in our moving coordinate system the motion is steady. Note that the influence of β is neglected because the parameter $\beta l/f$ (the ratio between the variations of the Coriolis parameter across the eddy and the Coriolis parameter at the center) is typically $\sim 0(0.01)$, so that its influence on the ring's shape is not larger than a few %. We shall see later that the effect of the environmental shear on the eddy's shape is much larger.

We shall focus our attention on frictionless and non-diffusive movements and assume that all the motions are hydrostatic. The governing equations for our moving coordinate system are obtained by applying the transformations $\hat{x} \to x + Ct$, $\hat{y} \to y$ to the time-dependent equations in a stationary coordinate system $(\hat{x}, \hat{y})$. For the conditions mentioned above, the governing equations are,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{2.1}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + f(u + C) = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{2.2}$$

$$\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0, \tag{2.3}$$

where u and v are the depth-independent [$u = u(x, y)$, $v = v(x, y)$] horizontal velocity components in the x and y directions, $h(x, y)$ is the eddy's total depth and p is the hydrostatic pressure.

Since the eddy is shallow (i.e., $h \ll l$, where h and l are the eddy's depth and length scales, respectively) and the lower layer is infinitely deep, the migration of the eddy has no effect on the fluid below (see e.g., Nof, 1981). In view of this, the pressures in the upper and lower layers are

$$p_1 = \rho g(\eta + \eta_0 - z),$$

and

$$p_2 = \rho g(\eta_0 - z). \tag{2.4}$$

Here η and η_0 are the free surface vertical displacements associated with the ring's motion and the environmental flow (respectively), g is the gravitational acceleration and, as mentioned before, ρ is the density of the eddy. The balance of forces in the lower environmental layer is,

$$f(U_0 + Ay) = \frac{-1}{\rho}\frac{\partial p_2}{\partial y} = -g\partial\eta_0/\partial y, \tag{2.5}$$

Where U_0 is the advecting layer speed at $y = 0$ and A is its shear.

Substitution of (2.5) and (2.4) into (2.1)–(2.2) gives,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g'\frac{\partial h}{\partial x}, \tag{2.6}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + f(u + C) = -g'\frac{\partial h}{\partial y} + f(U_0 + Ay), \tag{2.7}$$

where g' is the "reduced gravity" defined by $(\Delta\rho/\rho)$ g and $(\Delta\rho/\rho) \ll 1$ so that the interface displacement ξ $(=\eta\rho/\Delta\rho)$ can be approximated by h (i.e., $h = \eta + \xi \approx \xi$). These equations are similar to the usual shallow water equations except that they contain an additional pressure term $[f(U_0 + Ay)]$ corresponding to the pressure transmitted from the environmental fluid to the eddy.

The set (2.6)–(2.7) and (2.3) is subject to the boundary conditions:

$$h = 0, \quad \phi(x, y) = 0, \tag{2.8}$$

$$(\mathbf{i}u + \mathbf{j}v)\cdot\nabla h = 0 \quad \phi = 0, \tag{2.9}$$

where ∇ is the horizontal (two-dimensional) del-operator and ϕ denotes the ring's outer edge. Condition (2.8) states that $h = 0$ along a curve which is not known in advance (ϕ), and (2.9) requires that the edge will be a streamline. These conditions correspond to the fact that the location and shape of the ring's outer edge are not known *a priori* but rather must be determined as part of the problem. Note that since our model is frictionless, the velocity is not required to be continuous across $\phi(x, y) = 0$. This results from the fact that in an inviscid model, only the pressure should be continuous and there are no restrictions on the velocity (see e.g., Batchelor, 1967, Garabedian, 1964).

3. Solution

To obtain the solution to the problem, the governing equations will be simplified using a perturbation scheme in $\varepsilon = A/B$, the ratio betweeen the environmental shear (A) and the shear that the eddy would have in the absence of any mean flow (B). For this purpose, the following non-dimensional scaled variables are defined,

$$\left.\begin{aligned} &x^* = x/R_d, \quad y^*/R_d, \quad h^* = h/\hat{h}, \\ &\quad \varepsilon = A/B, \\ &u^* = u/BR_d, \quad v^* = v/BR_d, \quad R_0 = B/f, \\ &\quad C^* = C/BR_d \\ &U_0^* = U_0/AR_d \quad R_d = (g'\hat{h})^{1/2}/f. \end{aligned}\right\} \tag{3.1}$$

Here, R_0 is the ring's Rossby number [$\sim 0(1)$], R_d is the internal deformation radius, and $\hat{h}$ is the ring's maximum depth (at the center). The parameter ε is small for most rings of practical interest [$\sim 0(0.1)$] because the environmental shear is typically 0.1 m s^{-1}/100 km, whereas the eddie's shear is about 1 m s^{-1}/100 km.

In terms of these non-dimensional variables, the equations governing the ring's interior [(2.6), (2.7) and (2.3)] are:

$$R_0^2\left(u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*}\right) - R_0 v^* = -\frac{\partial h^*}{\partial x^*}, \tag{3.2}$$

$$R_0^2\left(u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}\right)+R_0(u^*+C^*)$$

$$=-\frac{\partial h^*}{\partial y^*}+R_0\varepsilon(U_0^*+y^*), \tag{3.3}$$

$$\frac{\partial}{\partial x^*}(h^*u^*)+\frac{\partial}{\partial y^*}(h^*v^*)=0, \tag{3.4}$$

where, as stated previously, $R_0 \sim O(1)$. It is further assumed that the dependent variables possess power series expansion in ε:

$$\left.\begin{array}{l} h^*=h^{(0)}+\varepsilon h^{(1)}+\ldots, \quad u^*=u^{(0)} \\ \quad +\varepsilon u^{(1)}+\ldots, \\ C^*=\varepsilon C^{(1)}+\varepsilon^2C^{(2)}+\ldots, \end{array}\right\} \tag{3.5}$$

where the zeroth-order state corresponds to a ring embedded in a resting ocean $[U_0=A=0]$.

By substituting (3.5) into (3.2), (3.3) and (3.4) one finds the zeroth-order equations,

$$R_0^2\left[u^{(0)}\frac{\partial u^{(0)}}{\partial x^*}+v^{(0)}\frac{\partial u^{(0)}}{\partial y^*}\right]$$

$$-R_0v^{(0)}=-\frac{\partial h^{(0)}}{\partial x^*}, \tag{3.6}$$

$$R_0^2\left[u^{(0)}\frac{\partial v^{(0)}}{\partial x^*}+v^{(0)}\frac{\partial v^{(0)}}{\partial y^*}\right]$$

$$+R_0u^{(0)}=\frac{-\partial h^{(0)}}{\partial y^*}, \tag{3.7}$$

$$\frac{\partial}{\partial x^*}[h^{(0)}u^{(0)}]+\frac{\partial}{\partial y^*}[h^{(0)}v^{(0)}]=0, \tag{3.8}$$

and the first-order equations,

$$R_0^2\left[u^{(1)}\frac{\partial u^{(0)}}{\partial x^*}+u^{(0)}\frac{\partial u^{(1)}}{\partial x^*}+v^{(1)}\frac{\partial u^{(0)}}{\partial y^*}\right.$$

$$\left.+v^{(0)}\frac{\partial u^{(1)}}{\partial y^*}\right]-R_0v^{(1)}=-\frac{\partial h^{(1)}}{\partial x^*}, \tag{3.9}$$

$$R_0^2\left[u^{(1)}\frac{\partial v^{(0)}}{\partial x^*}+u^{(0)}\frac{\partial v^{(1)}}{\partial x^*}+v^{(0)}\frac{\partial v^{(1)}}{\partial x^*}\right.$$

$$\left.+v^{(1)}\frac{\partial v^{(0)}}{\partial x^*}\right]+R_0[u^{(1)}+C^{(1)}]=-\frac{\partial h^{(1)}}{\partial y^*}$$

$$+R_0(U_0^*+y^*), \tag{3.10}$$

$$\frac{\partial}{\partial x^*}[h^{(1)}u^{(0)}+h^{(0)}u^{(1)}]+\frac{\partial}{\partial y^*}[h^{(0)}v^{(1)}$$

$$+h^{(1)}v^{(0)}]=0. \tag{3.11}$$

The zeroth- and first-order boundary conditions are:

$$h^{(0)}=0, \quad \phi^{(0)}(x^*,y^*)=0, \tag{3.12}$$

$$u^{(0)}\frac{\partial h^{(0)}}{\partial x^*}+v^{(0)}\frac{\partial h^{(0)}}{\partial y^*}=0, \quad h^{(0)}=0, \tag{3.13}$$

and

$$h^{(0)}+\varepsilon h^{(1)}=0, \quad \phi^{(0)}(x^*,y^*)+\varepsilon\phi^{(1)}(x^*,y^*)=0, \tag{3.14}$$

$$\left[u^{(0)}\frac{\partial h^{(0)}}{\partial x^*}+v^{(0)}\frac{\partial h^{(0)}}{\partial y^*}\right]+\varepsilon\left[u^{(1)}\frac{\partial h^{(0)}}{\partial x^*}\right.$$

$$\left.+u^{(0)}\frac{\partial h^{(1)}}{\partial x^*}+v^{(1)}\frac{\partial h^{(0)}}{\partial y^*}+v^{(0)}\frac{\partial h^{(1)}}{\partial y^*}\right]=0,$$

$$h^{(0)}+\varepsilon h^{(1)}=0. \tag{3.15}$$

Conditions (3.14) and (3.15) are somewhat complicated because the location and shape of the boundary are not known in advance. In general, the condition stated by the left-hand side of (3.15) should be satisfied only along the edge ($h^*=0$). We shall see, however, that the solution which will be shortly derived satisfies this condition everywhere (i.e., each of the terms in the square brackets is identically zero). This means that not only is the edge a streamline but every line of constant h^* is also a streamline.

By writing (3.6)–(3.8) in polar coordinates, it is easy to show that the zeroth-order system always possesses radially symmetric solutions. For simplicity, we shall follow the analysis of Nof (1981) and Killworth (1983) and choose a zeroth-order solution which corresponds to a linear orbital velocity:

$$u^{(0)} = y^*, \quad v^{(0)} = -x^*, \quad h^{(0)} = 1 - \tfrac{1}{2}R_0 \times (1 - R_0)[(x^*)^2 + (y^*)^2]. \tag{3.16}$$

The choice of such a basic state is acceptable for several reasons. Firstly, the largest velocity occurs along the ring's edge which is in agreement with various observations (see e.g., Csanady, 1979). Secondly, the depth distribution fits the observed structure of the 15 °C isotherm (Fig. 1) quite well and, thirdly, this basic state is known to be stable to small perturbations (Killworth, 1983). The velocity is discontinuous across the edge but, as pointed out earlier (Section 2), this is common in inviscid models (see e.g., Killworth, 1983, Killworth and Stern, 1982, Rossby, 1938). Note that the anticyclonic shear must be smaller than f so that $R_0 \leqslant \frac{1}{2}$; otherwise, the ring's relative vorticity would be larger than f and the eddy would be inertially unstable (see Nof, (1981)).

The solution of the first-order problem (3.9)–(3.15) is found by inspection, to be:

$$\left.\begin{aligned} &u^{(1)} = \frac{y^*}{R_0}, \quad v^{(1)} = 0, \quad C^{(1)} = U_0^*, \\ &h^{(1)} = \tfrac{1}{2}R_0(x^*)^2 + \tfrac{1}{2}(y^*)^2\,(2R_0 - 1), \end{aligned}\right\} \tag{3.17}$$

so that the total solution is,

$$\left.\begin{aligned} &u^* = y^* + \varepsilon y^*/R_0 + O(\varepsilon^2), \quad v^* = -x^* + O(\varepsilon^2) \\ &h^* = 1 - \tfrac{1}{2}R_0(1 - R_0)[(x^*)^2 + (y^*)^2] \\ &\quad + \tfrac{1}{2}\varepsilon R_0(x^*)^2 + \tfrac{1}{2}\varepsilon(y^*)^2\,(2R_0 - 1) \\ &\quad + O(\varepsilon^2), \\ &C^* = \varepsilon U_0^* + O(\varepsilon^2). \end{aligned}\right\} \tag{3.18}$$

Note that this solution is well behaved both along the edge and at the center even though $h^* = 0$ along the edge and $x^* = y^* = 0$ at the center. This results from the fact that perturbations themselves are small everywhere so that the expansion (3.5) is valid everywhere.

4. Discussion

The solution (3.18) has the following properties: (a) the horizontal velocity component in the x direction is modified by the shear (fyA/B), whereas the horizontal velocity component in the y direction remains unaltered; (b) the ring migrates at the mean environmental speed (U_0) and its edge is an ellipse; (c) the orientation of the ring does not rotate but rather remains fixed in space.

The dimensional lengths of the ellipse along the x and y axes are:

$$a = R_d\sqrt{2}\,[1 + fA/2B(f - B)]\,f/[B(f - B)]^{1/2} + O\left[R_d\left(\frac{A}{B}\right)^2\right], \tag{4.1}$$

$$b = R_d\sqrt{2}\left[1 + \frac{fA(2 - f/B)}{2B(f - B)}\right]f/[B(f - B)]^{1/2} + O\left[R_d\left(\frac{A}{B}\right)^2\right], \tag{4.2}$$

so that the ellipticity of the edge is,

$$\left(\frac{a}{b}\right) = 1 + fA/2B^2 + O(A/B)^2. \tag{4.3}$$

We see that, when $A > 0$ (anticyclonic shear), the major ellipse axis is oriented along the x axis (since B is always positive), whereas when $A < 0$ (cyclonic shear), the major axis is oriented along the y axis. The mean environmental speed (U_0) has no influence on the ring's shape; its only effect is on the ring's migration.

The various possible combinations of U_0 and A and their influence on the ring's structure and translation are shown in Fig. 3. The physical explanation for the resulting shapes and translations is fairly simple. To illustrate its characteristics, it is noted that, since the environmental flow is directed along the x axis, it affects only the u component (i.e., $v^{(1)} = 0$ as stated by (3.17)). In addition, it is recalled that only the shear affects the structure; the absolute value of the environmental flow has no influence on the ring's shape. Under such conditions, the exterior shear (Ay) simply increases ($A > 0$) or decreases ($A < 0$) the ring's u component, depending on its direction. When the environmental shear is anticyclonic, it adds to the u component of the anticyclonic eddy, whereas when it is cyclonic it opposes the eddy's circulation. Since the v component remains unaltered and the mass flux circulating within the ring is

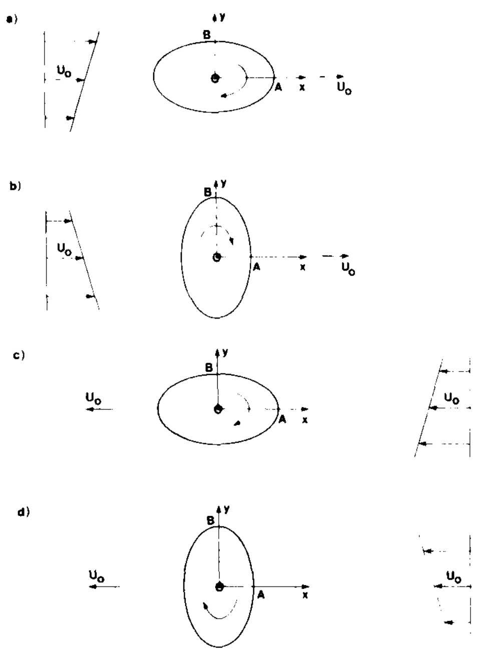

Fig. 3. Schematic diagram of the ring's structure in the presence of a mean flow with a linear shear. The ring translates at the mean environmental speed (i.e., the environmental speed at $y = 0$). For an anticyclonic environmental shear (cases (a) and (c)), the ellipse is oriented along the x axis whereas for a cyclonic environmental shear (cases (b) and (d)) the ellipse is oriented along the y axis. This behaviour results from the fact that an anticyclonic environmental shear increases the ring's u component, whereas a cyclonic environmental shear decreases the ring's u components.

constant, the distance between the eddy's center and its edge along the y axis must be either larger ($A < 0$) or smaller ($A > 0$) than the distance to the edge along the x axis. Consequently, the resulting shape is an ellipse which is oriented along the x or y axis.

These results are similar to those obtained by Moore and Saffman (1972) who, as mentioned earlier, investigated the behaviour of an inviscid barotropic vortex in a sheared flow on a *non-rotating* earth. In a similar fashion to our study, they found that when a vortex is placed in a sheared flow, its shape becomes elliptical. They also found that, as in our study, when the shear rotation is in the same sense as the vortex, the ellipse is parallel to the mean flow, whereas when the shear rotation opposes the vortex rotation, the ellipse is perpendicular to the mean flow. Since Moore and Saffman's study addresses a different problem (where stratification and rotation are unimportant) a detailed comparison between the two studies is meaningless and, therefore, is not presented.

We shall now apply our model to the warm-core rings found north of the gulf stream. The purpose of our application is not to convince the reader that the observed shape can only result from environmental shear but merely to point out that environmental shear is a possible cause of ring ellipticity. The rings, in the region in question, are embedded in a sheared flow resulting from the fact that the gulf stream flows eastward, whereas the slope water is flowing westward (see e.g., Hansen, 1977, Lai and Richardson, 1977). Due to the limited number of available measurements, it is difficult to determine the environmental speed along the line connecting the ring's center and the axis of the environmental flow. In view of this, we shall not attempt to compare the predicted advection speed to observations, but rather focus our attention on the influence of the shear on the ring's shape.

Fig. 4 shows a typical observation of a warm-core ring (courtesy of O. Brown and B. Evans). We see that the ring's shape resembles an ellipse whose major axis is in the N–S direction, as predicted by our model (Fig. 3d). In addition to this particular observation, other observations of the same ring (Brown *et al.*, 1983) suggest that, as a whole, the ring does not rotate much; namely, the major axis remains in the N–S direction most of the time. This property is also predicted by our simplified model because it gives a stationary pattern (in the moving coordinate system).

While these general agreements are informative, a more detailed comparison is necessary for our results to be meaningful. Due to the simplifications involved in our analysis, a completely satisfactory comparison is impossible. We shall demonstrate below, however, that our model gives the correct order of magnitude of the ring's relative dimensions. To show this, it is noted that in the area under discussion, the environmental shear is roughly -10^{-6} s^{-1}. This estimate is obtained by considering the speed of the westward flowing slope water which is typically ~ 5 cm s^{-1} (see e.g., Hansen, 1977) and a deformation radius of ~ 50 km. Note that a shear of $O(10^{-6})$ s^{-1} is usually found in many parts of the open ocean (see e.g., Elliott and Sanford, 1984) so that it is a reasonable estimate for the shear in question. The ring's shear is

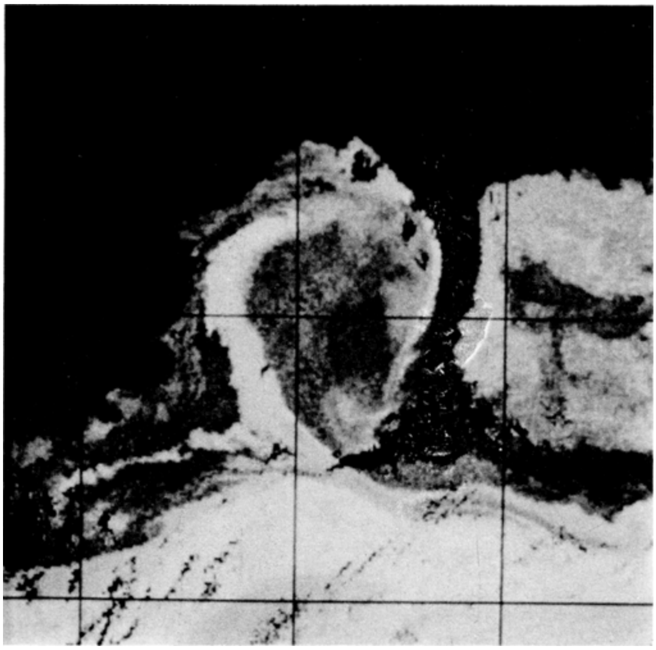

Fig. 4. Atmospherically corrected NOAA-7 AVHRR infrared image of a typical warm-core ring (courtesy of O. Brown and R. Evans). Note that the ring's shape is similar to an ellipse oriented in the N–S direction as predicted by the model. The ring is centered at approximately 40° N and 64° W. The grid is 2° × 2° latitude longitude on a navigated image remapped to a mercator projection. Image was processed by the University of Miami, Remote Sensing Center.

roughly 1.5 m s^{-1}/100 km (see e.g., Csanady (1979), Cheney (1976)), so that $B \sim 1.5 \times 10^{-5}$ s^{-1}. For these values and $f \sim 10^{-4}$ s^{-1}, relation (4.3) gives,

$$\frac{a}{b} \approx 0.78,$$

suggesting that the major ring's axis (in the N–S direction) is ~20% larger* than the minor axis as shown in Fig. 5. This value agrees qualitatively with the ellipticity displayed by Fig. 4 as well as the ellipticity of the ring at other times (see Fig. 2a of Brown et al. (1983)), supporting the idea that the ellipticity of the ring may result from the environmental shear. It is quite possible that the ellipticity could also be a result of waves propagating along the ring perimeter. Such a mechanism was

* Note that the chosen numerical values give a relatively large value for the perturbed velocities, but that the error in the computation of the edge shape is smaller than 5%.

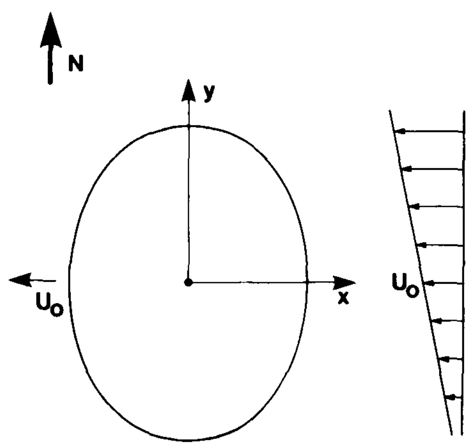

Fig. 5. A diagram of the predicted shape of a warm-core gulf stream ring. The elliptical shape results from the cyclonic environmental shear (see text). Here $A \approx -10^{6}$ s^{-1}, $B \approx 1.5 \cdot 10^{-5}$ s^{-1} and $f \approx 10^{-4}$ s^{-1} so that (by (4.3)) the ratio between the minor and major axes is ≈0.78. This figure should be compared to Fig. 4 which displays a similar ellipticity.

suggested by Spence and Legeckis (1981) for cyclonic cold-core rings. However, the fact that the warm-core rings observed north of the gulf stream do not rotate much (Brown et al., 1983) suggests that the application of their mechanism may be limited to cold-core rings.

5. Summary

Apart from demonstrating that the non-linear structure of lens-like rings embedded in a sheared flow can be computed analytically, our results give important information on the behaviour of warm-core rings. Before listing our conclusions, it is appropriate to mention again the limitations involved in the analysis. The most important assumptions which have been made throughout the analysis are that the ring is frictionless and non-diffusive, that it has a lens-like cross section and that the lower layer is infinitely deep. Flierl et al. (1983) and Flierl (1984) have pointed out that, on a β plane, the last assumption may be quite restrictive. It is not clear that this would also be the case with the problem in question (which addresses eddies on an f plane), but it seems that this aspect deserves further investigation. The results of the study can be summarized as follows:

(i) a lens-like ring embedded in a flow with a linear shear translates at the mean environmental speed;

(ii) due to the environmental shear, the eddy's shape is an ellipsoid whose major axis is either parallel or perpendicular to the mean flow; for an anticyclonic environmental shear, the major ellipse's axis is oriented along the environmental flow, whereas for a cyclonic shear it is oriented perpendicular to the mean flow;

(iii) the ratio between the ellipse's dimensions along the x and y axes is given by $(1 + fA/2B^2)$, where A is the environmental shear and B is the ring's shear along the x axis;

(iv) the ring as a whole does not rotate due to the environmental shear so that the ellipse's orientation remains fixed.

Application of (i)–(iv) to the warm-core gulf stream rings suggests that the observed ellipticity may result from the environmental shear. For these rings, the model predicts an elliptical shape with a ratio of 120% between the major (N–S) and minor (E–W) axes.

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