

The Mindanao and Halmahera eddies – twin eddies induced by nonlinearities

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Abstract

Starting with the simple case of a nonlinear northward (southward) western boundary current (WBC) flowing along a concave solid boundary with a sharp corner on a β -plane, it is shown analytically that an anticyclonic (cyclonic) eddy is established to balance the upstream momentum flux. (On an f -plane no eddy is established; similarly, no eddy is established in the linear limit.) The solution is derived by expanding the dependent variables in powers of $\epsilon^{1/6}$ for the northward current (anticyclonic eddy) and $\epsilon^{1/2}$ for the southward current (cyclonic eddy). Here $\epsilon = \beta R_d / f_0$, where R_d is the Rossby radius. It is found that, for a high Rossby number (i.e., highly nonlinear current), the length scales are $R_d / \epsilon^{1/6}$ and $R_d / \epsilon^{1/2}$ for the anticyclonic and cyclonic eddies (respectively) implying that these eddies are larger than most eddies in the ocean (which scale with R_d). The anticyclonic eddy scale is the same as that recently obtained for a WBC separating from the wall (Arruda et al. 2002) but the cyclonic eddy scale has not been obtained before. A reduced gravity numerical model is used to validate these analytical results. The balance of forces and the eddy size estimates (derived from the numerical simulations) agree with the analytical results.

With the aid of the above analysis we then examine the collision of two opposing western boundary currents on a β -plane. It is shown that this problem can be conceptually reduced to the above problem of two WBCs turning inside a solid corner on a β -plane (where the streamline separating the two colliding currents acts like a zonal wall). We show that an eddy is established on each side of the dividing streamline. Based on the above collision problem, it is argued that the Halmahara and Mindanao eddies are required to balance the nonlinear momentum fluxes of their colliding parent currents, the southward flowing Mindanao Current (MC) and the northward flowing South Equatorial Current (SEC).

1. Introduction

The western equatorial Pacific plays a key role in the establishment of El Niño and Southern Oscillation (e.g., Webster and Lukas 1992) and may also be an important part of the so-called “great conveyor belt”(e.g., Gordon 1986) because of the Pacific-to-Indian throughflow (ITF). The eddies and low-latitude western boundary current (WBC) addressed here (**Fig. 1**) are important aspects of these processes.

The equatorward-flowing WBCs provide the closure to two symmetrical gyres relative to the equator (Kessler and Taft 1987). One gyre is entirely in the Northern Hemisphere whereas the other (which is mostly in the Southern Hemisphere) crosses the equator. The North Equatorial Counter-Current (NECC) forms the boundary between these two gyres at about 5°N. North of the NECC the westward-flowing North Equatorial Current (NEC) bifurcates (Toole et al. 1990) into the northward-flowing Kuroshio and the southward-flowing Mindanao Current (MC). A similar situation takes place in the Southern Hemisphere where the westward-flowing South Equatorial Current (SEC) bifurcates (around 15°S) into a branch flowing northwestward and a branch flowing southward. Along the New Guinea coast the northwestward-flowing branch of the SEC is usually recognized as a subsurface current, the New Guinea Coastal Under-Current (NGCUC), and a surface current, the New Guinea Coastal Current (NGCC). The NGCC retroflects to the east of the Halmahera Island and joins the retroflected flow of the MC to flow eastward as the NECC.

There are two semi-permanent eddies in the retroflection area of the MC and SEC (**Fig.1**). The first, the Mindanao eddy (ME), is situated north of the NECC (near 7°N, 128°E) and has cyclonic circulation, whereas the second, the Halmahera eddy (HE), which is situated south of the NECC (near 4°N, 130°E) has anticyclonic circulation (Wyrski 1961). Why the eddies established is not *a priori* clear. One would intuitively expect that such eddies are established by friction which enables the turning fluid to drag some interior fluid along with it as it turns. Another possibility would be that the eddies result from instability of the eastward flowing NECC and the westward propagation of such instabilities. We shall show in this article

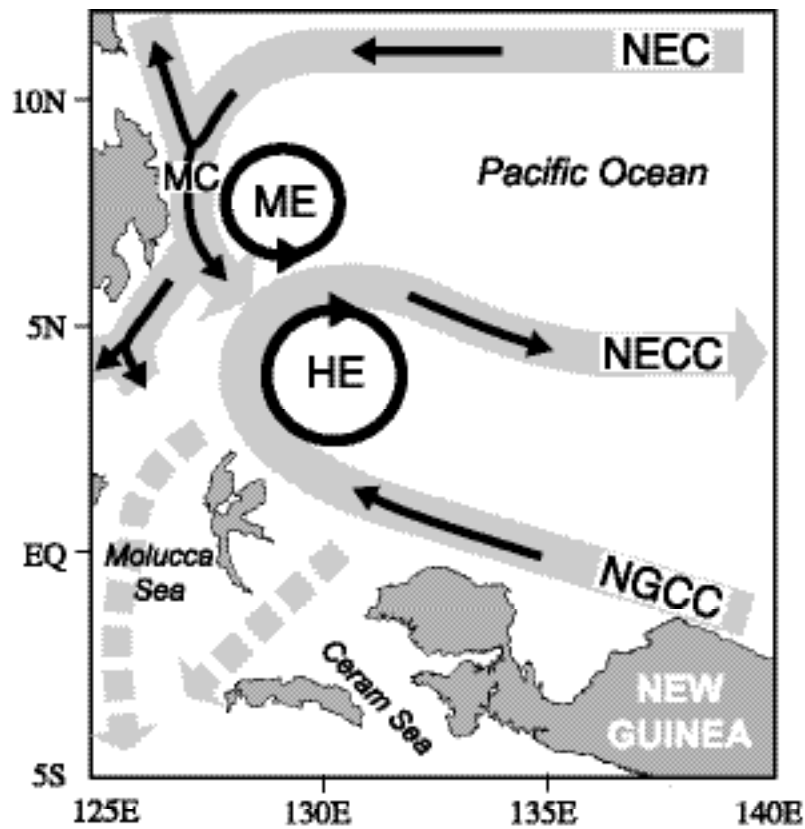


Fig. 1. The flow pattern in the western equatorial Pacific (adapted from Field and Gordon 1992). The Mindanao eddy (ME) and the Halmahera eddy (HE) are semi-permanent and do not usually drift away from their generation area. Dashed arrows denote the southern part of the Indonesian throughflow (ITF).

that neither of the two ideas is correct. We shall demonstrate that both friction and instability do not play any role in the establishment of the eddies. Rather, it is nonlinearity and β which are responsible for the establishment of the eddies. In what follows we shall briefly describe the two colliding boundary currents (MC and SEC) and the two eddies.

a. *The Mindanao Current (MC)*

The MC extends to a depth of 600 m and a distance of 100 km offshore (Wyrтки 1961, Masuzawa 1968). Wyrтки (1956, 1961) first estimated a baroclinic transport ranging from 8 to 12 Sv ($1 \text{ Sv} = 10^6 \text{ m}^2 \text{ s}^{-1}$) in the upper 200 m and 25 Sv in the upper 1000 m. A somewhat different estimate was later made by Masuzawa (1969) who gave a transport from 13 to 29 Sv relative to 600 dbar, Kendall (1969) who argued that the MC carries 14 Sv, Cannon (1970) who estimated a geostrophic transport of 18 to 31 Sv relative to 1000 dbar, Toole et al.(1988) who gave a transport of 17-18 Sv for waters warmer than 12°C , and Lukas et al.(1991) who calculated a transport from 13 Sv to 33 Sv between 10°N and 5.5°N . Like many other WBCs, the relatively high velocities [$O(1) \text{ m s}^{-1}$] and the fairly narrow width (150 km) give a fairly high Rossby number (~ 0.5 taking into account that the maximum speed is at that jet's center) suggesting that the current is nonlinear.

b. *The Mindanao eddy (ME)*

Takahashi (1959) first noted the existence of "a cold region of distorted elliptic form" east of the MC and related this feature to the cyclonic circulation inferred from dynamic topography. This closed circulation is named "Mindanao eddy" following the work of Wyrтки (1961) who noted that the ME is a quasi-permanent eddy associated with the turning of the NEC waters at the coast of the Philippines and their subsequent flow to the east as part of the NECC. The existence of the eddy was later verified by Lukas et al. (1991) who reported that drifters launched in the ME described closed loops with diameters of about 250 km and by Qu et al.

(1999) who identified the ME as a depression (of less than 130 m) in the $24.5 \sigma_\theta$ isopycnal surface centered at 7°N , 129°E .

c. *The South Equatorial Current and the New Guinea Coastal Current*

As mentioned, the westward-flowing SEC bifurcates near 15°S . The equatorward branch of the Great Barrier Reef Undercurrent (Church and Boland 1983) later flows into the NGCUC. A shallow current (NGCC) overlying the NGCUC was first observed by Masuzawa (1968). This shallow current is the current that we are interested in. Cantos-Figuerola and Taft (1983) found an NGCC transport of 11 Sv whereas Wyrтки and Kilonsky (1984) estimated a total transport (NGCC plus NGCUC) of about 40 Sv. Gouriou and Toole (1993) found a total transport of 24.8 Sv from direct measurements, 37.7 Sv from geostrophy relative to 600 dbar, and 41.7 Sv from geostrophy relative to 1000 dbar. Like the MC, the NGCC is fairly nonlinear with a Rossby number of approximately 0.4.

d. *The Halmahera eddy (HE)*

As with the ME, the HE appears in the dynamics topography maps of Takahashi (1959) and was named after the work of Wyrтки (1961). It is well developed only during the summer monsoon, when the South Pacific water from the NGCC re-curves into the NECC. Within the HE, Lukas' et al.(1991) drifters executed closed loops of about 300 km diameter and velocity of about 50 cm s^{-1} . Using shipboard ADCP, Kashino et al.(1999) identified the center of the Halmahera eddy to be east of 130°E . They also identified a horizontal scale of about 500 km (at 50 m). Using data from drifters released in the HE during summer and fall of four different years, Waworuntu (1999) showed that the HE has large relative vorticity suggesting that its core contains mainly south Pacific water.

e. *Present work*

As mentioned, our goal is to examine the nonlinear collision of opposing WBCs on a β -plane. We shall see that it is the curving of these retroflecting currents and β which are responsible for the generation of the eddies. Before proceeding, it is appropriate to point out that the Indonesian Throughflow (ITF) is critical to the collision of the MC and the NGCC and, therefore, to the establishment of the eddies. Arruda (2002) showed that, without any net meridional flow a few degrees north of the equator (i.e., no ITF), there would be no WBC transport and, hence, no collision and no eddies. This is consistent with the picture described in Nof (2000) where, due to the vanishing wind stress curl a few degrees north of the equator, a no-ITF scenario must involve no net WBC transport (i.e., no collision).

This paper is organized as follows. After presenting the method of analysis in Section 2, we address the problem of a WBC in a concave solid corner in Section 3. In Section 4 we focus on the collision problem, and in Section 5 we apply the results of the previous sections to the equatorial western Pacific and suggest the physical mechanism responsible for the existence of the ME and HE. The conclusions are given in section 6.

2. **Methods**

Since our problem involves nonlinearity, a “head-on” approach is not useful and we shall look at the problem in terms of integrated momentum flux balances which circumvent the need to find a solution valid in the entire field. The approach is similar to that of a single WBC separation (Arruda et al. 2002) and it is easier to introduce it by briefly reviewing this work. Consider a single northward flowing WBC in an upper layer (with density ρ) above an infinitely deep layer of slightly denser water (with density $\rho + \Delta\rho$). As the current flows northward it reaches a latitude where the upper layer thickness vanishes on the wall (due to an outflow to the

east) and, consequently, separation takes place (**Fig.2**).

Assuming steady state and integrating the nonlinear inviscid y-momentum equation over the region *ABCD* (**Fig. 2**), it was shown by Arruda et al.(2002) that,

$$\int_0^L h v^2 dx = \beta \int_{eddy} \psi dx dy, \quad (1)$$

where L is the boundary current width, u and v are the velocity components in the x and the y direction, h is the upper layer thickness, and ψ is the transport streamfunction defined by $\partial\psi/\partial y = -uh$; $\partial\psi/\partial x = vh$. (Our notation is conventional and is described in both the text and the Appendix.)

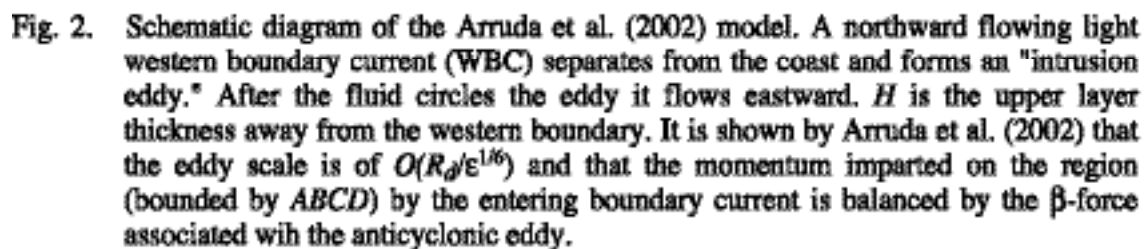
From (1) we can immediately see that the eddy is established in order to balance the upstream (northward) momentum flux of the WBC. In this scenario the presence of β produces a southward (eddy) force balancing the northward momentum flux imparted by the separating WBC. This shows that, in contrast to the familiar idea attributing the formation of eddies to instabilities (i.e., the breakdown of known steady solution), the “intrusion eddy”¹ is an integral part of the steady stable solution. On an f -plane [i.e., the right-hand side of (1) vanishes] no eddy is formed and in the linear limit [i.e., the left-hand side of (1) is zero] there is no eddy either. Equation (1) is similar to that used by Nof and Pichevin (1999) to estimate the size of the Tsugaru and Alboran gyres but each of the terms is very different leading to an eddy size which is quite different from the gyre’s size.

Using expansions in $\epsilon^{1/6}$ (where $\epsilon = \beta R_d / f_0$ and R_d is the Rossby radius), Arruda et al. (2002) derived a lower bound for the eddy radius,

$$R = 2 \frac{\hat{E}_2^{5/2} \epsilon^{1/6}}{\hat{E}_5 p} \sim \frac{R_d}{\epsilon^{1/6}}. \quad (2)$$

The small parameter $\epsilon \propto \beta R_d / f_0$ is typically $O(10^{-3})$ so that $\epsilon^{1/6} \sim O(0.3)$. This implies that the

¹ This term was first used by Olson (1991) to describe an eddy associated with a western boundary current separation.



intrusion eddy is usually larger than most of the eddies associated with instabilities (which scale with R_d). Our approach to the collision problem will be similar to that described above, i.e., we shall integrate the equations over region containing the collision.

3. Flow in a concave solid corner

a. Analytical solutions

Before attacking the full collision problem it is useful to first examine the behavior of currents in a concave solid corner formed by a solid boundary. We shall do so by using the momentum flux approach introduced above and begin by examining a *northward* flowing current.

1. Formulation As the WBC (**Fig. 3**) flows northward it encounters a zonal wall that forces it to flow eastward. Assuming a steady state and integrating (after multiplying by h) the steady and inviscid nonlinear y -momentum equation over the fixed region S bounded by the dashed line $ABCD$ (shown in the upper left panel in **Fig. 3**), we get,

$$\oint_S \left(\frac{\partial(huv)}{\partial x} + \frac{\partial(hv^2)}{\partial y} \right) dx dy - \oint_S u \frac{\partial \psi}{\partial y} dx dy - \oint_S \frac{\partial(\beta y \psi)}{\partial y} - \beta \psi dx dy + \frac{g'}{2} \oint_S \frac{\partial(h^2)}{\partial y} dx dy = 0, \quad (3)$$

and application of Green's theorem gives,

$$\oint_{\partial S} h u v dy - \oint_{\partial S} h v^2 + \frac{g' h^2}{2} - (f_0 + \beta y) \psi dx + \beta \oint_S \psi dx dy = 0, \quad (4)$$

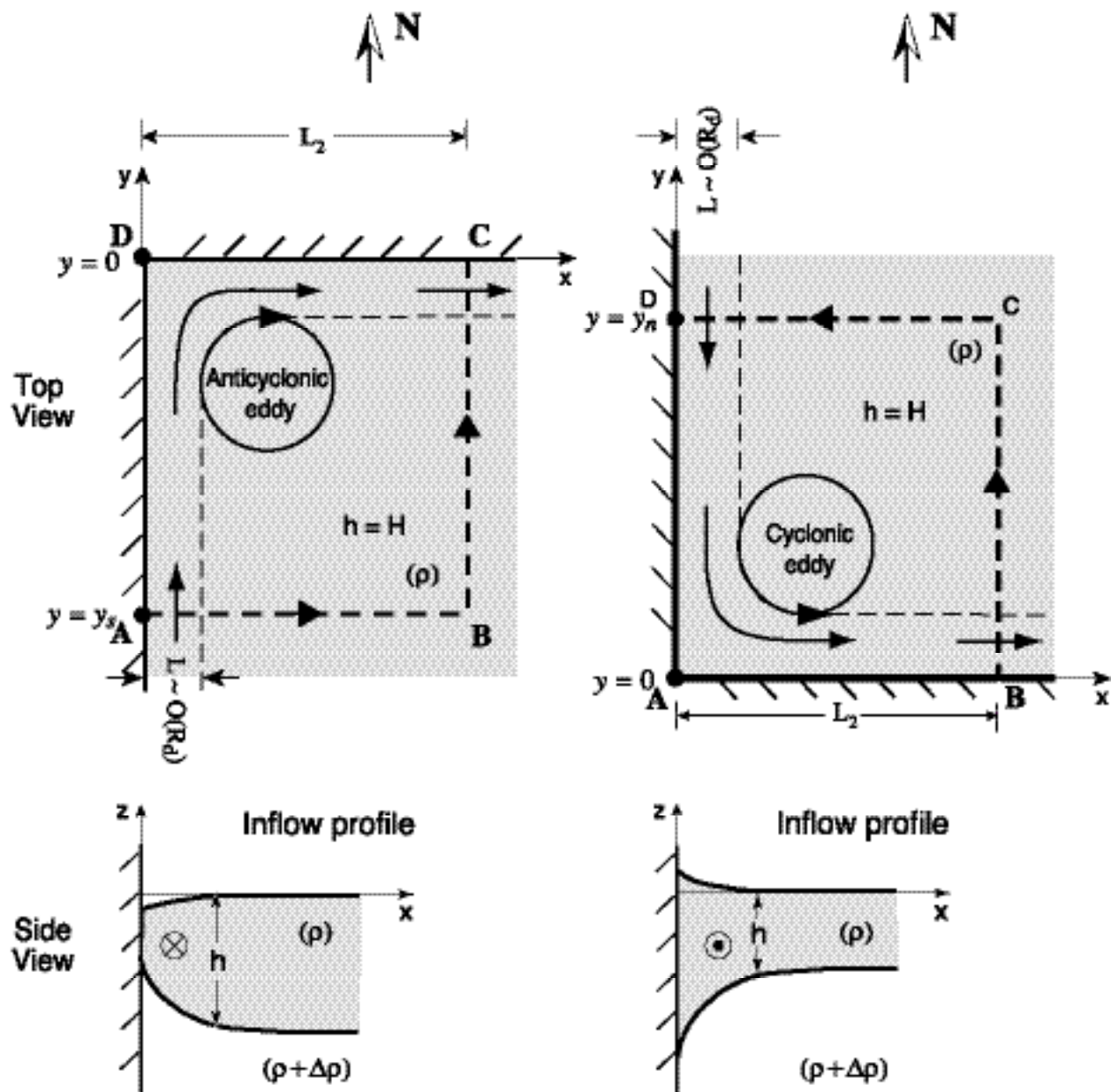


Fig. 3. Schematic diagram of the solid corner model. *Upper panels:* A northward (southward) flowing western boundary current (WBC) encountering a zonal wall. H is the upper layer far east from the western boundary on a latitude a few Rossby radii away from the zonal wall. *Lower panels:* Vertical cross-section of the approaching northward (southward) flowing WBC.

where ∂S is the boundary of S and g' is the “reduced gravity,” $gD\rho/\rho$.

Next, we take $\psi = 0$ along the wall and note that at least one of the velocity components vanishes on every portion of the boundary ∂S . It then follows from (4) that,

$$\oint_{\partial S} \left[h v^2 + \frac{g' h^2}{2} - (f_0 + \beta y) \psi \right] dx - \oint_C \frac{g' h^2}{2} dx - \beta \oint_S u v dx dy = 0. \quad (5)$$

Assuming now (and later verifying with our numerics) that, away from the corner, the flow is geostrophic in the cross-current direction, we get [after multiplying the geostrophic relation $(f_0 + \beta y)v = g'\partial h/\partial x$ by h and integration in x],

$$(f_0 + \beta y)\psi = [h^2 - h(0, y_s)^2]. \quad (6)$$

Combining (5) and (6) we get our desired expression,

$$\boxed{\underbrace{\oint_0^L h v^2 dx}_{\text{WBCmom.flux}} + \underbrace{\frac{g'}{2} \oint_0^{L_2} (h^2(0, y_s) - h^2(x, 0)) dx}_{\text{Pressure term}} - \underbrace{\beta \oint_S u v dx dy}_{\beta \text{ term}} = 0,} \quad (7)$$

where L is the boundary current width and L_2 is the zonal extent of our region S . It is straightforward to show that, for southward-flowing WBC, the equivalent momentum balance in the region S (bounded by $ABCD$) is very similar.

2. The f -plane limit. On an f -plane ($\beta \neq 0$) the pressure force should balance the WBC momentum flux if a steady state is to be established. To see this, note that, as we approach the corner, the velocity along the wall gradually decreases to zero (see Kundu 1990, Chapter 4). Since the wall is a streamline, the Bernoulli function [$B = g'h + (u^2 + v^2)/2$] implies that the upper layer thickness increases to a maximum at the corner (**Fig. 4**, upper panel). Consequently, the pressure force in (7) points in the opposite direction to that of the WBC momentum force and a

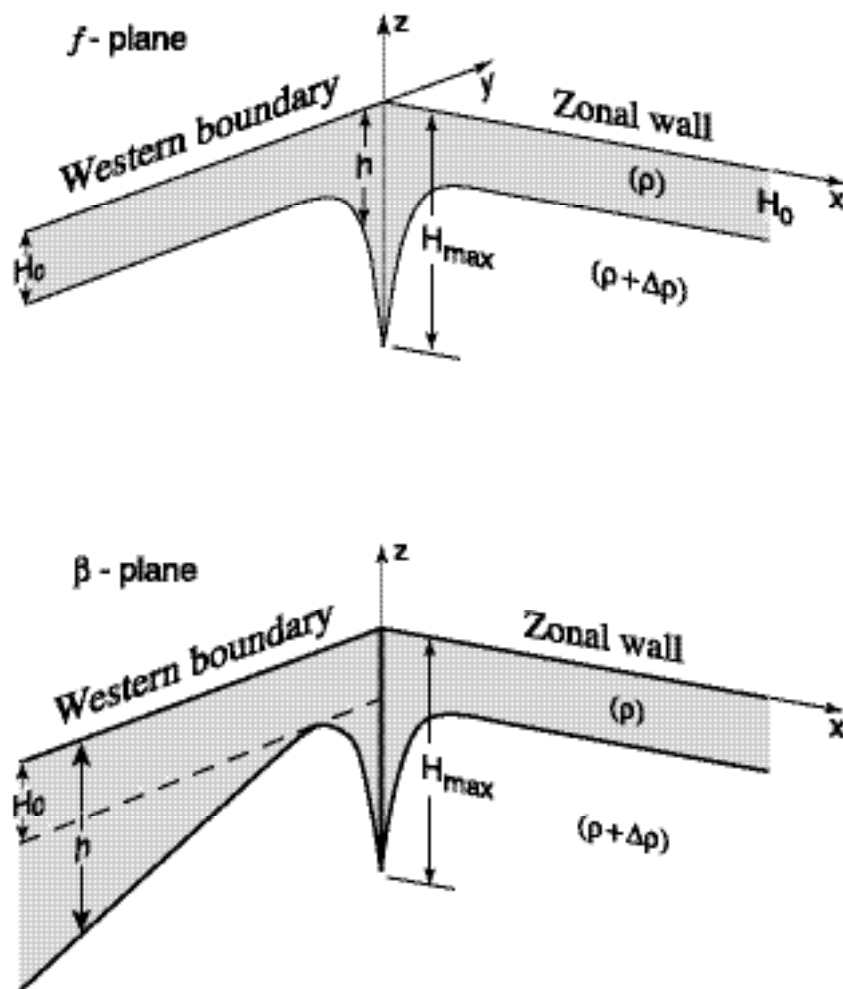


Fig. 4. Schematic three-dimensional diagram of the upper layer thicknesses on the boundaries of a northward flowing western boundary current on an f -plane (upper panel) and on a β -plane (lower panel). Note that the upstream near wall thickness is greater in the β -plane case. This is why the eddy is established (see text).

balance without an eddy appears to be possible. Numerical simulations will later verify this outcome. Note that, in the case of no zonal wall (i.e., the WBC separates due to a vanishing upper layer thickness), the pressure term vanishes (since $h = 0$ on the western boundary and on the outcropping streamline). As a result, the WBC momentum flux is unbalanced and the f -plane system *cannot* reach a steady state (see Arruda et al. 2002).

3. The no eddy on a β -plane scenario. Here, we temporarily assume that no eddy is associated with the turning boundary current on a β -plane and show that this hypothetical scenario is impossible. Note that the geostrophic transport relationship $[T = g'(H^2 - h_w^2)/2f]$, where H and h_w are the thicknesses off and on the wall] implies that on a β -plane the near-wall thickness of a northward-flowing WBC decreases as we proceed downstream along the western boundary (**Fig. 4**, lower panel). This implies that there is an additional pressure force (resulting from the decreasing near-wall thickness due to β) pointing in the same sense as the upstream momentum flux. Hence, the wall pressure cannot balance the net (upstream) northward force. Taking the zonal extent of S to be $O(R_d)$ we see that, in the absence of an eddy, the β -term in (7) is negligible compared to the WBC momentum flux so that it cannot balance the WBC momentum flux either. The *no-eddy* scenario is, therefore, *impossible*.

b. Numerical simulation for a flow in a concave solid corner

We use a reduced gravity version of the isopycnic model developed by Bleck and Boudra (1981,1986) and later improved by Bleck and Smith (1990). This model is suitable for our study because it allows isopycnic outcropping by using the "Flux-Corrected Transport" algorithm (Boris and Book 1973; Zalesak 1979) in the continuity equation. The equations of motion are the two momentum equations,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - (f_0 + \beta y)v = -g \frac{\partial h}{\partial x} + \frac{v}{h} (h - u)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} - (f_0 + \beta y)u = -g \frac{\partial h}{\partial y} + \frac{v}{h} (h - v) ,$$

and the continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = 0 ,$$

where v is the frictional coefficient.

The model uses the Arakawa (1966) C-grid where the u -velocity points are shifted one-half gridstep to the left from h points, the v -velocity points are shifted one-half gridstep down from the h points, and vorticity points are shifted one-half gridstep down from the u -velocity points. On open boundaries the Orlanski (1976) second-order radiation boundary condition was implemented. The list of experiments is given in Table 1. The walls were slippery and the vorticity was taken to be zero near them.

1. Northward flowing WBC in a concave corner. **Fig. 5** shows the upper layer thickness and streamfunction for the f -plane experiment E1 and the β -plane experiment E5 (Table 1) at day 2500, and **Fig. 6** shows the upper layer thickness along the western boundary and along the zonal wall. We see that, as mentioned, on an f -plane the upper layer thickness increases to a maximum near the corner, producing a southward pressure force. For the f -plane experiment E1 the downstream upper layer thickness along the zonal wall is 270.7 m which is very close to our specified value of 270.3 for H_0 . **Fig. 7** shows that the pressure force points southward and balances the northward momentum flux of the WBC [indicating that the inviscid balance is attained]. We also see that, as mentioned, in this f -plane case, no eddy is necessary to achieve the momentum balance.

When β is introduced (**Fig. 5**), an anticyclonic eddy (attached to the curving flow) is generated. As seen in **Fig. 6**, in this case, the net pressure force points northward since the upper layer thickness along the western boundary is larger than its average value on the zonal wall

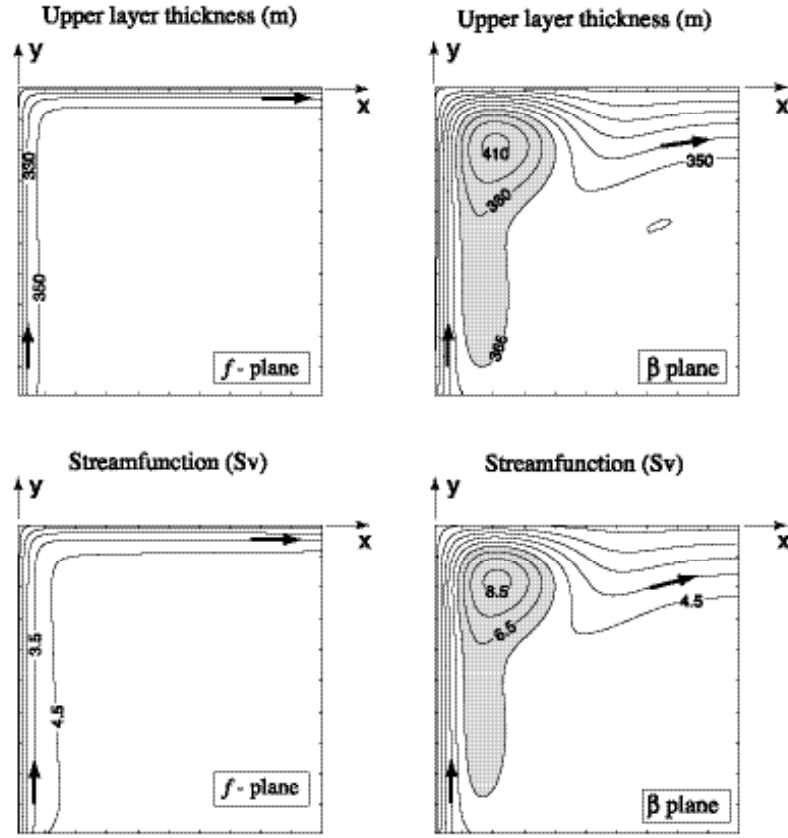


Fig. 5. Upper layer thickness contours (upper left) and streamfunction contours (lower left) for the f -plane experiment E1 (Table 1) at day 2500; upper layer thickness contours (upper right) and streamfunction contours (lower right) for the β -plane experiment E3 (Table 1) at day 25000. The contour spacing is 20 m for the upper layer thickness and 15 Sv for the streamfunction. The axis marks are 10 grid points (75 km) apart. Areas within the closed contours are shaded.

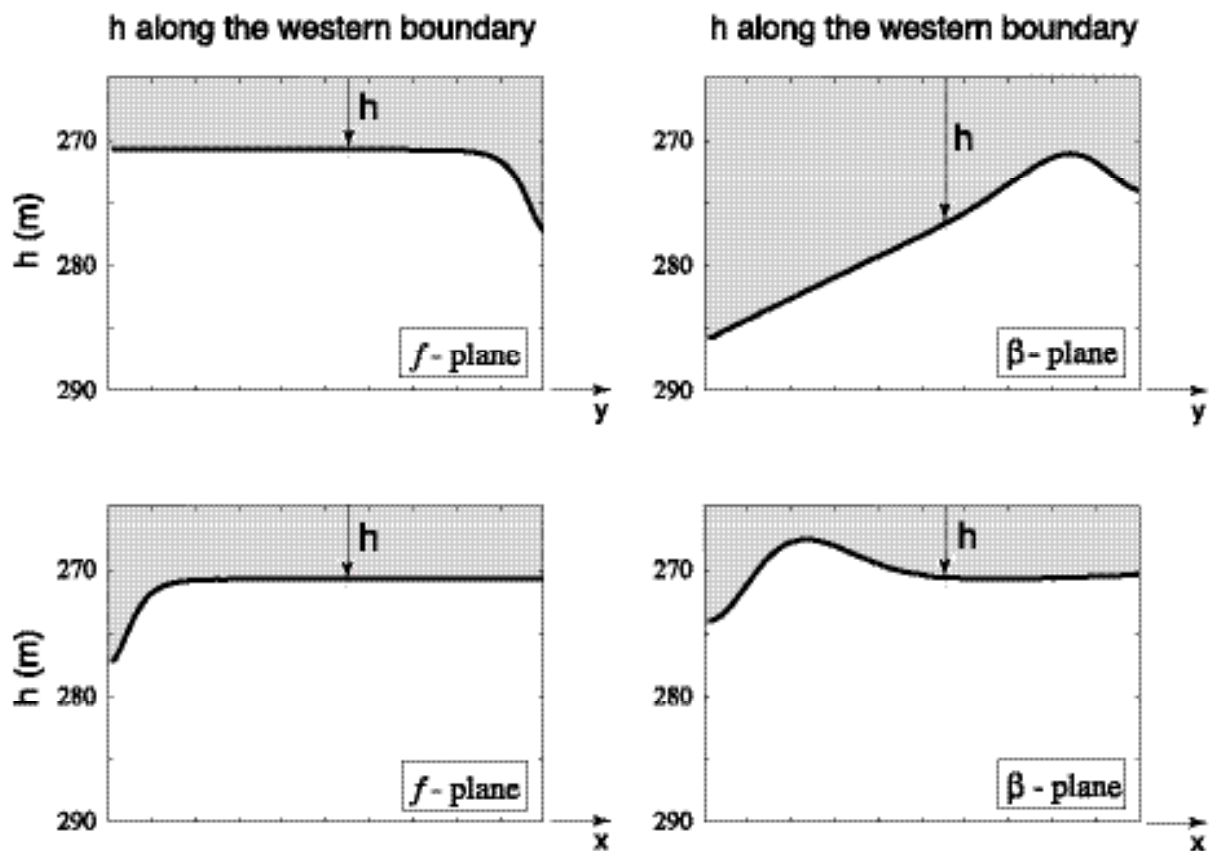


Fig. 6. The f -plane upper layer thickness h (in meters) along the western boundary (upper left) and along the zonal wall (lower left) at day 2500; β -plane upper layer thickness h (in meters) along the western boundary (upper right) and along the zonal wall (lower right) at day 25000. Shaded area is the upper layer. The horizontal axis marks are 10 grid points (75 km) apart.

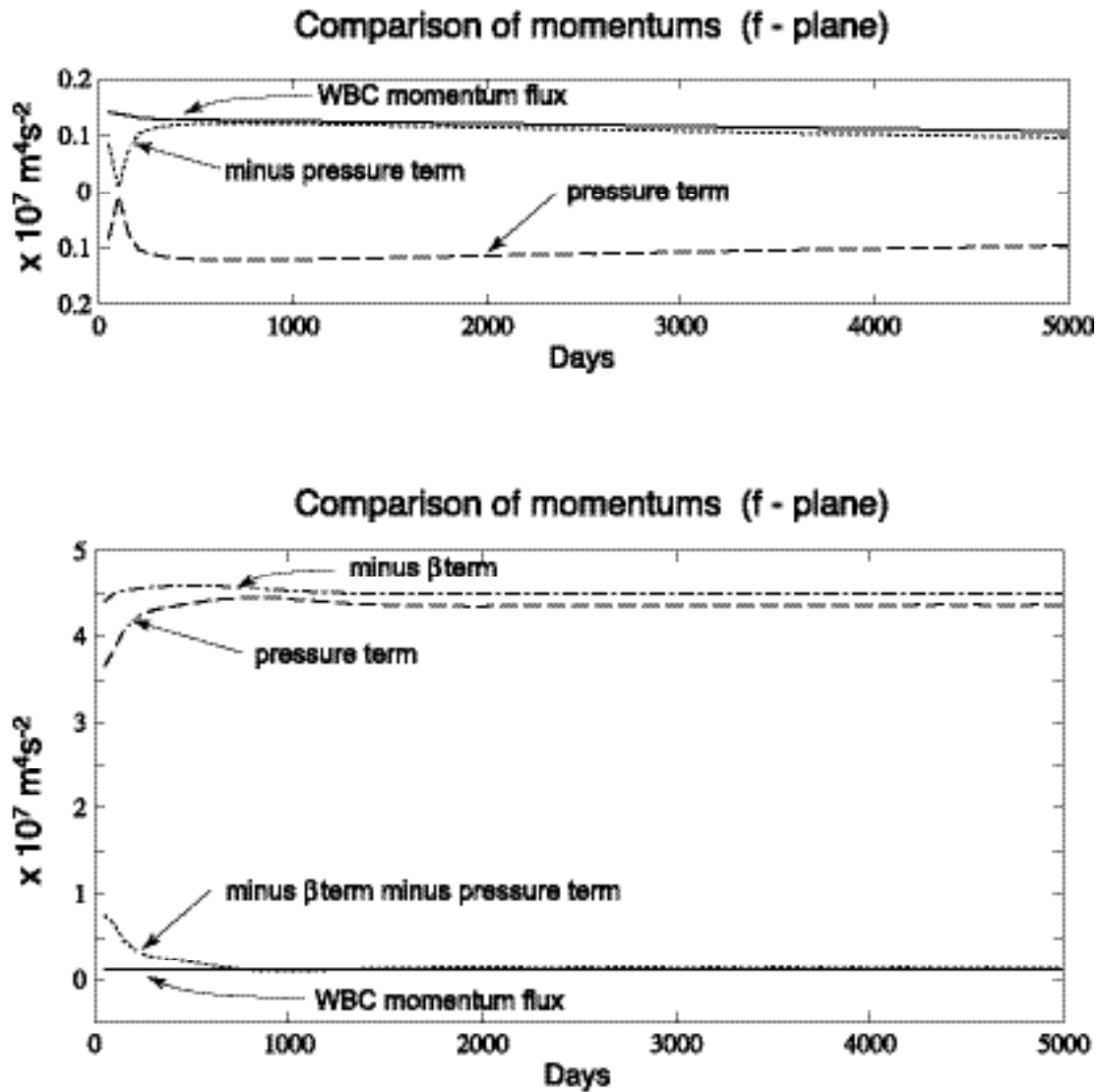


Fig. 7. Terms of the momentum balance (9) computed from the f -plane experiment E1 (upper panel, and the β -plane experiment E3 (lower panel). Note that, by day 500 for the f -plane run and day 800 for the β -plane run, the steady state balance holds.

(270.2m). In addition, note that the thickening of the upper layer at the corner is compensated by the eddy so that the average value of the upper layer thickness on the zonal wall is 270.2 m, which is practically indistinguishable from the initial value for H_0 (270.3 m). This numerical observation will be used shortly in our detailed derivations. **Fig. 7** shows that the combination of β and the pressure terms is a southward force which balances the northward momentum force of the WBC. With the aid of (7) and a scale analysis that we shall perform later, we shall show that the eddy is the main contributor to the combined β and pressure terms. The eddy's β -force is due to the particles circulation within the anticyclonic eddy which causes a greater Coriolis force on the northern portion of the eddy than on the southern.

To investigate the role of friction in the β -plane simulations we ran experiments E3 and E7 (not shown) with lower and higher frictional coefficients. In the first case, the system did not reach an steady state while in the second case friction smoothed out the eddy, making it weaker and meridionally elongated.

2. Southward flowing WBC in a corner. **Fig. 8** shows contour plots of the upper layer thickness and streamfunction at day 2500 for the f -plane experiment (E2) and the β -plane experiment (E6). Similarly, **Fig. 9** displays the upper layer thickness along the western boundary and along the zonal wall at the same day. In the f -plane situation, the upper layer thickness increases to a maximum in the corner, producing a northward pressure force which, according to **Fig. 10**, balances the southward momentum force of the WBC. This indicates that the inviscid balance is attained.

When β is introduced (**Fig. 8**), a cyclonic eddy is formed. Also, as seen in **Fig. 9**, the pressure force points southward since the upper layer thickness along the western boundary is larger than its average value on the zonal wall (268.6 m). The average value of the upper layer thickness along the zonal wall is 268.6 m which coincides with its average value at the last 50

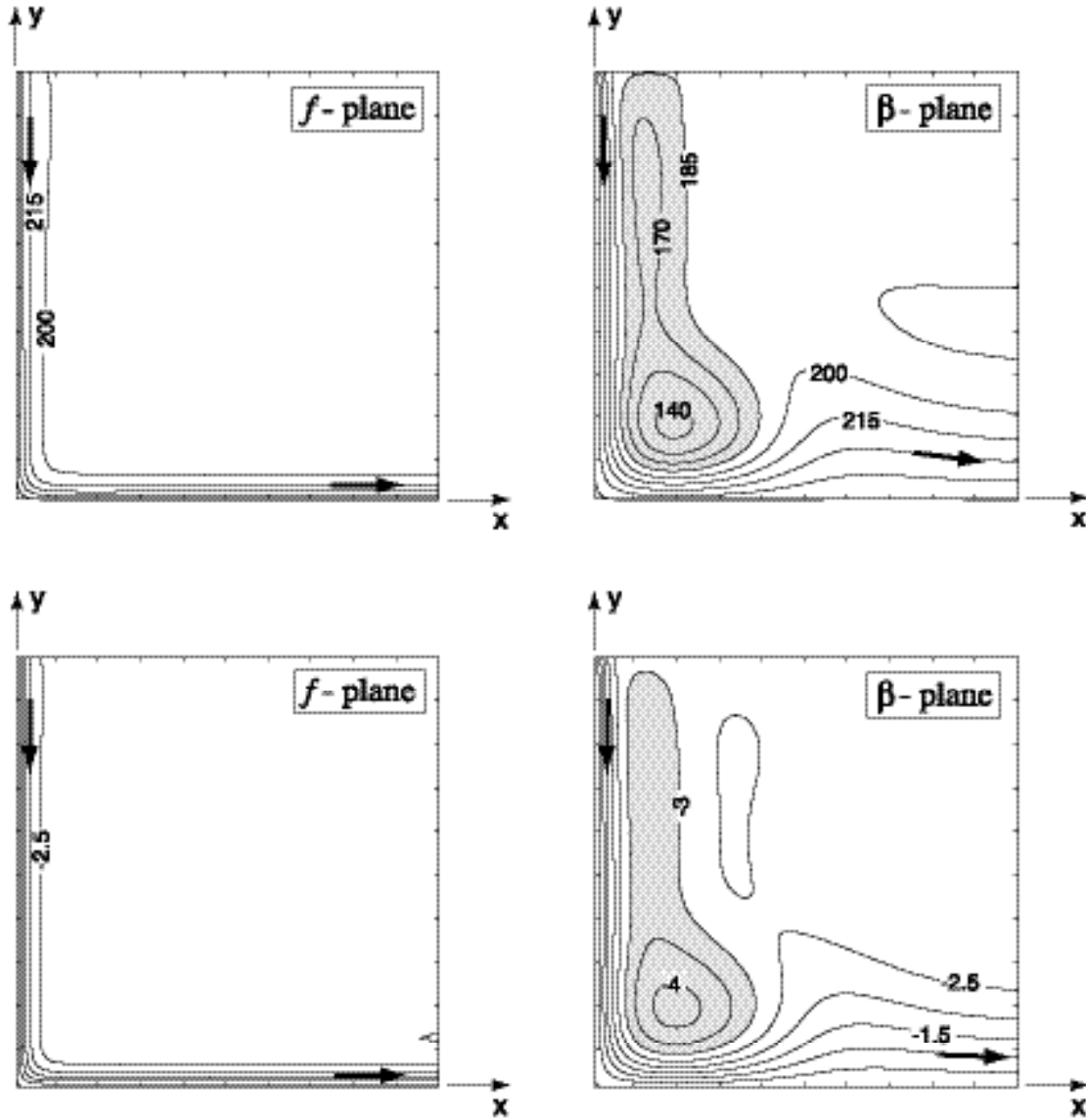


Fig. 8. Upper layer thickness contours (upper left) and streamfunction contours (lower left) for the f -plane experiment E2 (Table 1) at day 2500; upper layer thickness contours (upper right) and streamfunction contours (lower right) for the β -plane experiment E4 (Table 1) at day 25000. The contour spacing is 20 m for the upper layer thickness and 15 Sv for the streamfunction. The axis marks are 10 grid points (75 km) apart. Areas within the closed contours are shaded. Note the diagonal symmetry of the f -plane runs and the diagonal asymmetry of the β -plane runs.

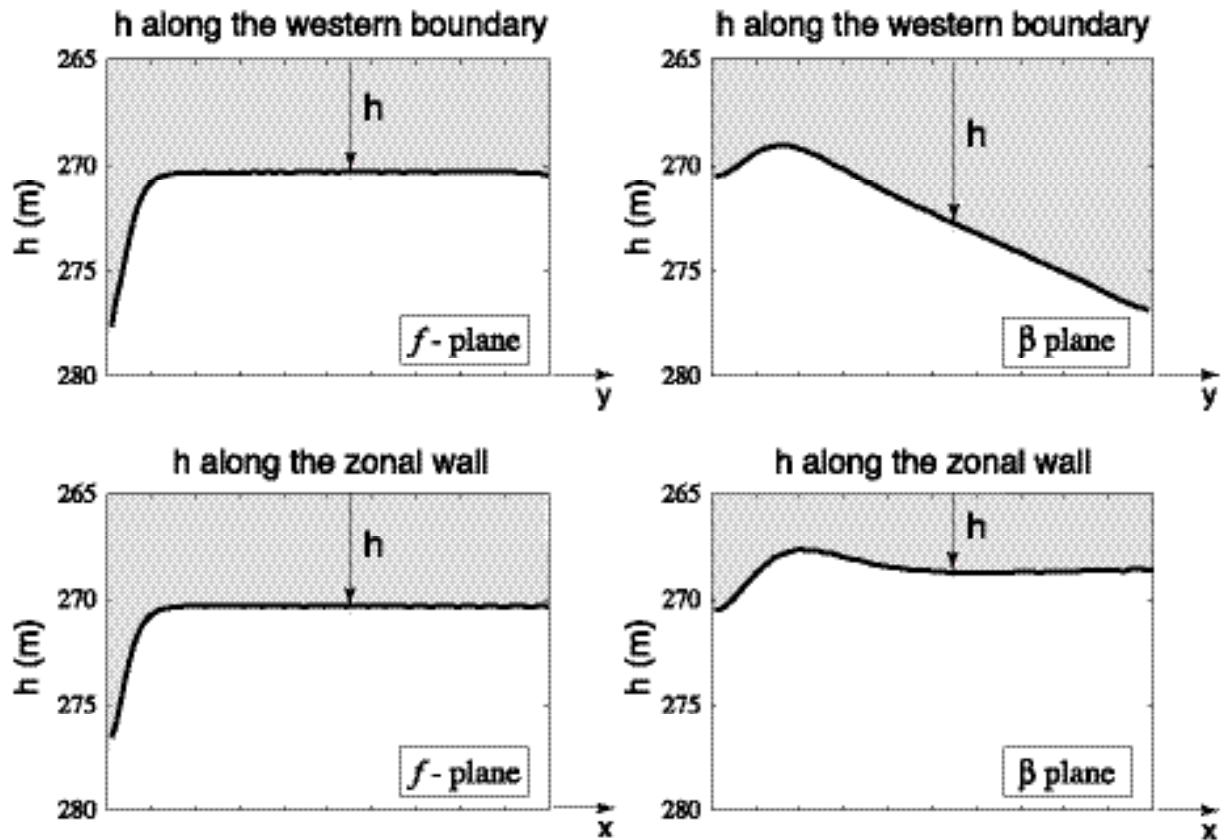


Fig. 9. The f -plane upper layer thickness h (in meters) along the western boundary (upper left) and along the zonal wall (lower left) at day 2500; β -plane upper layer thickness along the western boundary (upper right) and the zonal wall (lower right) at day 25000. Shaded area is the upper layer. The horizontal axis marks are 10 grid points (75 km) apart.

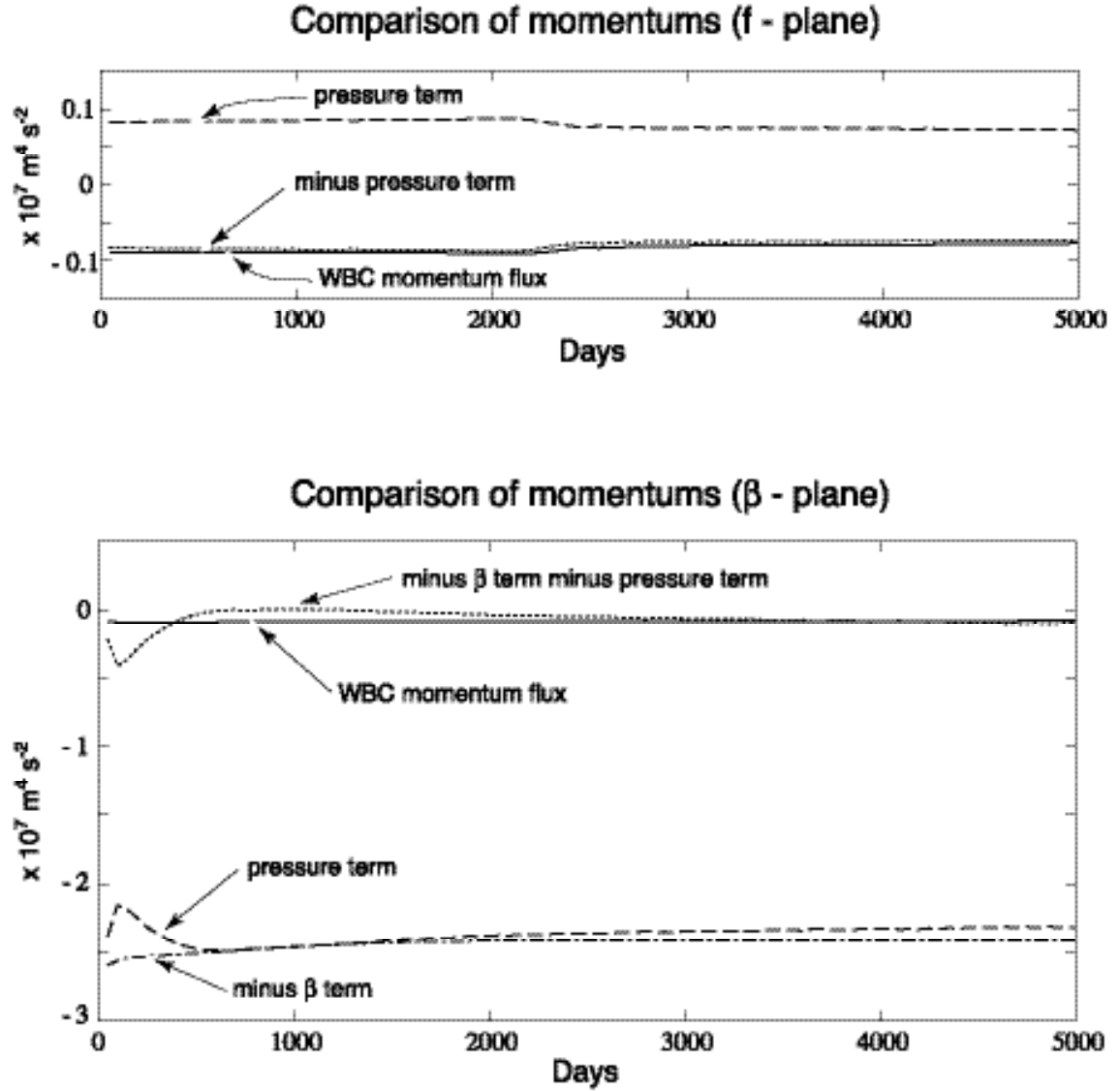


Fig. 10. Terms of the momentum balance (13) computed from the f -plane experiment E2 (upper panel), and the β -plane experiment E4 (lower panel). Note that the f -plane run reaches a steady state right away [within a period of $O(f^{-1})$] while the β -plane run takes 500 days to reach a steady state.

grid points, showing that the thickening at the corner (associated with the Bernoulli function conservation) is compensated by the shallowness produced by the eddy. Again, this will shortly be used in our derivations. As seen in **Fig. 10**, the combination of the β and the pressure terms is a northward force which balances the southward momentum force of the WBC.

c. Estimates of the eddy radius

We shall now use both the analytical consideration given in (a) and the numerical simulation given in (b) to derive an estimate for the eddy radii. An alternative form of the momentum balance relation for northward-flowing WBC in a concave solid corner (5) can be derived by noting that, at $x \rightarrow \infty$, the zonal flow is geostrophic in the y direction so that (5) can be written as,

$$\oint_C h v^2 dx + \frac{g}{2} \oint_C (H_0^2 - h^2(x, 0)) dx - \beta \oint_C \psi dy = 0 \quad (\text{northward WBC}) \quad (8a)$$

It is straightforward to show that, for a southward flowing WBC,

$$- \oint_C h v^2 dx + \frac{g}{2} \oint_C [h^2(x, 0) - H_0^2] dx - \beta \oint_C \psi dy = 0 \quad (\text{southward WBC}) \quad (8b)$$

In the next two subsections we shall use (8a,b) to derive analytical expressions for the eddy radius. We shall treat the cases of northward and southward flowing WBC separately (because the scales are different).

1. Northward flowing WBC and the anticyclone. As pointed out earlier, the numerical simulations indicate that the average value of the upper layer thickness on the zonal wall is approximately H_0 (the value of h on the zonal wall as $x \rightarrow \infty$). Although we cannot come up with any argument explaining why this should be so, we shall use this information for our calculations and neglect the second term in (8a). Recall that, on an f -plane, the pressure term is negative and

that a neglect of a negative term would overestimate the eddy size. We shall assume here that this is also true on a β -plane implying that our estimate will be a lower bound on the eddy size. With the above neglect, the momentum balance (8a) reduces to,

$$\int_0^L h v^2 dx = \beta \int_S \psi dxdy, \quad (9)$$

where L is the boundary current width, $\hat{\psi} = \psi - \psi_*$ and ψ_* is the streamfunction at $x = 0$.

Next, we take the following scales:

$$L \sim O(R_d); DH = (H - H_0) \sim O(H); v \sim O\left[(g'H)^{1/2}\right];$$

$$\hat{\psi} \sim O\left[\frac{\hat{E} g'H^2}{f_0^2}\right] \text{ in the current; and } \hat{\psi} \sim O\left[\frac{\hat{E} g'H_e^2}{f_0^2}\right] \text{ in the eddy,}$$

where $R_d = (g'H)^{1/2}/f_0$ and H_e is the thickness scale for the eddy. Note that the second term in (8a) is nonzero only in the eddy and the boundary current (because the streamfunction ψ coincides with ψ_* in the interior). Following Nof and Pichevin (1999) we now take $R_{de} = R_d/\varepsilon^{1/6}$ where $\varepsilon \equiv \beta R_d/f_0$ and obtain the leading order balance,

$$\int_0^{L^{(0)}} h^{(0)} (v^{(0)})^2 dx^* = \int_{S_e} \psi_e^{(0)} dx_e^* dy_e^*, \quad (10)$$

where $h^{(0)}$, $v^{(0)}$, $\psi_e^{(0)}$, and $L^{(0)}$ are the zeroth-order approximations of the respective dependent variables.

Next, we take the flow to have zero potential vorticity and find that the solutions are straightforward despite the nonlinearity. Note that, with this assumption, the interior of the basin is motionless (with thickness H) and the velocity is zero along the bounding streamline of the current offshore (see e.g., Anderson and Moore 1979). As we shall see later, in this limit, the

obtained estimate is a lower bound on the eddy radius. The leading order velocity and thickness for a zero-potential vorticity northward flowing boundary current are,

$$v = \begin{cases} f_0(L-x) & , \quad x \leq L \\ 0 & , \quad x > L \end{cases} \quad (11)$$

$$h = \begin{cases} H - \frac{f_0^2(L-x)^2}{2g'} & , \quad x \leq L \\ H & , \quad x > L, \end{cases} \quad (12)$$

where $L = 2^{1/2} R_{de}$, where $R_{de} = [g'(H - H_0)]^{1/2}/f_0$. Since the eddy's upper layer thickness scale is $H_e = H(R_{de}/R_d)^2 = \epsilon^{-1/3}H$, we can take $h_e = 0$ along the eddy's boundary and find that for the zero potential vorticity eddy, (i.e., $v_0 = -f_0 r/2$), (8–10) gives,

$$R_{anti} = 2 \frac{\hat{E}^{3/2} \epsilon^{1/6}}{\hat{E} p} \sim \frac{\hat{E} H - H_0}{\hat{E} H} \epsilon^{1/2} \frac{\hat{E} H}{\hat{E} H - H_0} - \frac{3}{5} \epsilon^{1/6} \frac{R_d}{\epsilon^{1/6}}. \quad (13)$$

This relation is a lower bound, because the zero potential vorticity eddy has the strongest nonlinearity (due to the highest steepness) and, consequently, it also has the smallest radius.

To validate (13) we determine analytically the numerical eddy radius using the parameters of the numerical experiment E5 (Table 1), getting $\epsilon^{1/6} = 0.42$ and $R = 2.84 R_d$. Taking the 6.5 Sv streamline as the eddy boundary (**Fig. 5**) we find that $3.7 R_d$ is the average numerical radius between days 1500 and 5000, which is in very good agreement with the above analytical estimate ($2.84 R_d$). In the next subsection we shall derive the radius estimate for the cyclonic eddy formed by a southward flowing WBC in a concave solid corner on a β -plane. We shall see that the scales of the cyclone are quite different from the scale of the anticyclone.

2. Southward flowing WBC and the cyclone. We proceed in a similar fashion to that of the previous section, pointing out that in this case the eddy upper layer thickness scales with H , so that the obtained eddy size scale is $R_d/\epsilon^{1/2}$ rather than $R_d/\epsilon^{1/6}$. A zero potential vorticity flow

is impossible here and we take the zeroth-order (basic state) boundary current and eddy to have uniform potential vorticity. We shall use the Csanady (1979) solution for uniform potential vorticity cyclonic eddy (**Fig. 11**). Next, the current is expanded around the f -plane solution with potential vorticity depth H and we ultimately find,

$$R_{cyclone} = \frac{\hat{\epsilon}^{1/2}}{\bar{\epsilon}^3 p} \frac{\hat{\epsilon} H_0 - H}{H} - \frac{\hat{\epsilon} H + 2 H_0}{H} \frac{\hat{\epsilon}^{1/2}}{\bar{\epsilon}} \frac{R_d}{\epsilon^{1/2}}. \quad (14)$$

As in the anticyclonic case, (14) is *lower bound* because the uniform potential vorticity eddy outcrops along a contour of radius $\bar{R}$ (where the streamfunction reaches the minimum value of $-g'H^2/f_0$).

Again, to validate (14) we estimate the eddy radius using the parameters of the numerical experiment E6 (Table 1), obtaining $\epsilon^{1/2} = 0.06$, $R_0 = 1.92R_d$, and $R = 3.35R_d$. Taking the -3.5 Sv streamline as the eddy boundary in E6 (**Fig. 8**), we estimate $4.28R_d$ as the average numerical radius between days 1500 and 5000. This is in a fair-to-good (though not very good) agreement with the analytical estimate ($3.35R_d$).

4. The collision problem

In this section we will examine the full collision problem of two opposing WBCs on a β -plane (**Fig. 12**). Recall that, as in the earlier case mentioned above, β is crucial here as, for an analogous f -plane situation, Agra and Nof (1993) showed that net momentum flux of the colliding currents is balanced, i.e., no eddies are necessary for the f -plane momentum balance to hold.

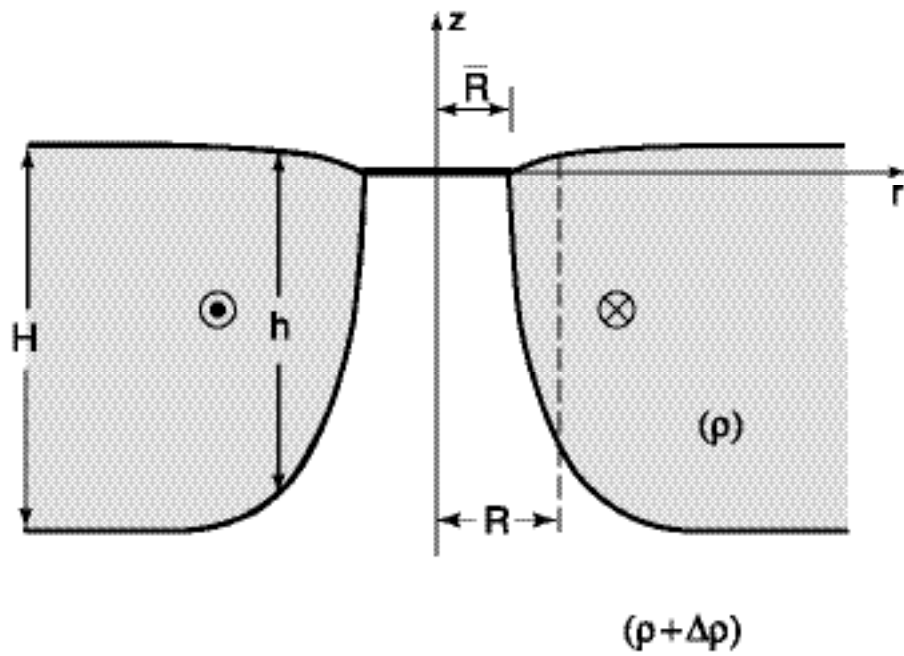


Fig. 11. Schematic representation of our cyclonic eddy, $\bar{R}$ is the radius of the outcropped region and R is the eddy mean radius, H is the undisturbed thickness at $r \rightarrow 0$.

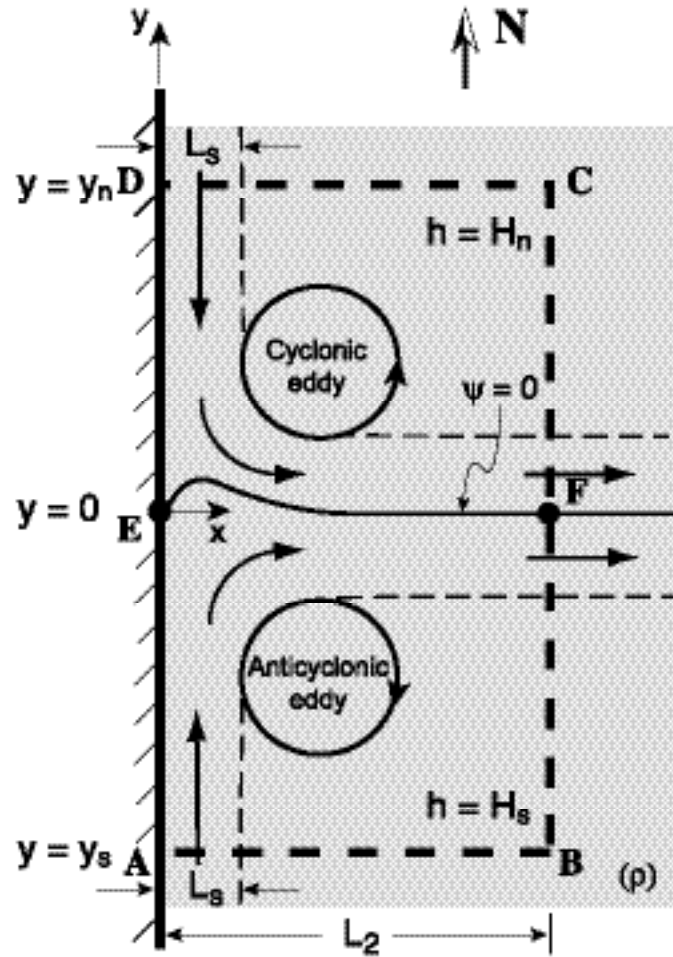


Fig. 12a. Schematic top view of the collision of a northward flowing boundary current (main current) and a southward flowing boundary current (counter current) on a β -plane. After the collision the two currents merge into a joined offshore current. East of the western boundary and a few Rossby radii away from the dividing streamline ($\psi = 0$) the upper layer thickness is H_s and H_n (where the subscripts "s" and "n" denote "south" and "north").

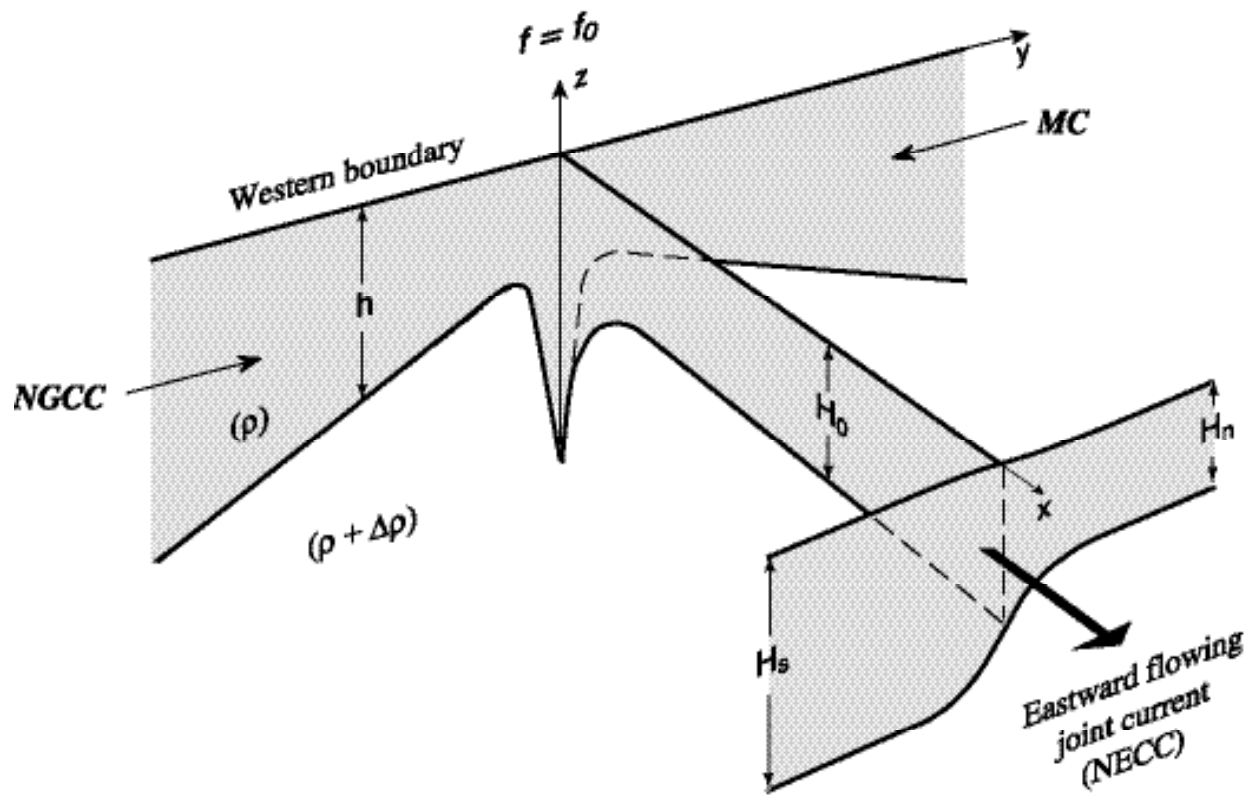


Fig. 12b. A three-dimensional view of the long wall and long-jet thicknesses.

a. *Formulation*

We shall denote the northward flowing current as the "main current" and the southward flowing one as the "counter current." At some point on the western boundary the currents collide and veer offshore as a joined current. We place the origin of our coordinate system at the collision (stagnation) point on the western boundary and assume that, far east of the western boundary and a few Rossby radii away from the dividing streamline the upper layer thickness has a value of H_s in the main current (south) side and H_n in the counter current (north) side. Assuming a steady state and integrating (after multiplying by h) the steady, inviscid nonlinear y-momentum equation over the fixed region S bounded by $ABCD$ we get,

$$\boxed{\underbrace{\int_0^{L_s} h v^2 dx - \int_0^{L_n} h v^2 dx}_{\text{Net momentum flux}} + \underbrace{\frac{g'}{2} \int_0^{L_2} [h^2(0, y_s) - h^2(0, y_n)] dx}_{\text{Pressure term}} - \underbrace{\beta \int_S \psi \psi dx dy}_{\beta \text{ term}} = 0,} \quad (15)$$

where L_s and L_n are the main current and counter current widths, S is the integration region bounded by $ABCD$ (**Fig. 12**), L_2 is the zonal width of S , and y_s and y_n are the y-coordinates of the southern and northern boundaries of S .

To apply our previous analytical approach to the collision problem, we divide the integration region into two sub-domains S^+ and S^- , north and south of $\psi = 0$, respectively.

$$\int_0^{L_s} h v^2 dx + \frac{g'}{2} \int_0^{L_2} [H_0^2 - h^2(0, \phi)] dx - \beta \int_{S^+} \psi (\psi - \psi_0) dx dy = 0, \quad (16)$$

and,

$$\int_0^{L_n} h v^2 dx + \frac{g'}{2} \int_0^{L_2} [h^2(0, \phi) - H_0^2] dx - \beta \int_{S^-} \psi (\psi - \psi_0) dx dy = 0, \quad (17)$$

where H_0 is the upper layer thickness on $\psi = 0$ as $x \rightarrow \infty$, and ψ_0 (a function of y only) is the

limit of ϕ as $x \rightarrow \infty$. As expected, the two have a mutual term $\frac{g'}{2} \oint h^2(x, \phi) dx$, where $y = \phi(x)$ is a Cartesian representation of the curve $\phi = 0$. We shall see in the next subsection that the second term in (14) and (15) is approximately zero and that, consequently, our solid corner solutions will also be valid here.

b. Numerical simulations

The parameters for the collision experiment E9 on a β -plane (Table 1) are identical to those used for experiments E5 and E6 for the concave corner. **Fig. 13** shows contour plots of the upper layer thickness and streamfunction for experiment E9 at day 2500. It is evident that an anticyclonic eddy is formed south of the joined offshore current and cyclonic eddy north of it. **Fig. 14** shows the upper layer thickness along the western boundary (upper panel) and along the $\psi = 0$ streamfunction (lower panel). Note the similarity with **Figs. 6 and 9** for the β -plane experiments E5 and E6. The pressure force points in the same direction as the momentum flux (in each of the individual sub-domains). Therefore, eddies are necessary in order to reach the integrated momentum balances in S^- and S^+ individually. The average value for the upper layer thickness on the zero streamfunction is 270.8 m while the mean value outside the eddy influence area is 271 m (**Fig. 14**), showing that, even in the collision problem, the condition that the second term in (16) and (17) vanishes is valid.

Fig. 15 shows the numerical estimates for the terms in (15) (upper panel), (16) (middle panel), and (17) (lower panel). In all the cases the inviscid momentum balances hold. The analytical estimates for the flow inside a concave corner can also be used here, by considering the $y = 0$ streamfunction as a “wall” dividing the basin in two parts (the main current side and the counter current side). Taking the -4 Sv streamline as the boundary of the cyclonic eddy and the 6.5 Sv streamline as the boundary of the anticyclonic eddy (**Fig. 13**) we estimate (from the numerics) an average radius of $4.5 R_{dn}$ [where $R_{dn} = (g'H_n)^{1/2}/f_0$] and $3 R_{ds}$ [where $R_{ds} = (g'H_n)^{1/2}/f_0$]. These are in decent agreement with the analytical estimates of $3.35 R_{dn}$ and $2.84 R_{ds}$, respectively. Recall that these analytical estimates neglect the second terms in (16) and (17).

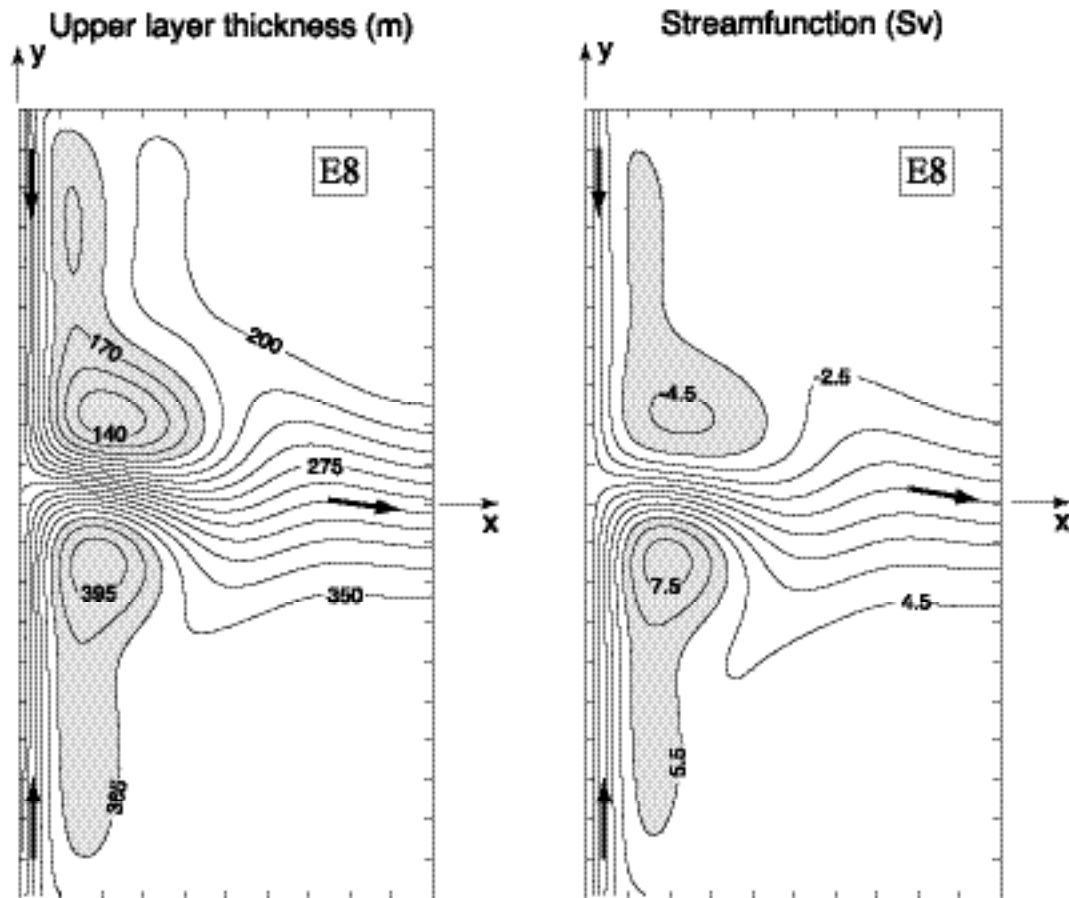


Fig. 13. Upper layer thickness contours (left panel) and streamfunction contours (right panel) for the collision experiment on a β -plane at day 2500. The axis marks are 10 grid points (75 km) apart. Areas within closed contours are shaded. Recall that no eddies are produced in the f -plane case (see Fig. 5 in Lebedev and Nof 1997).

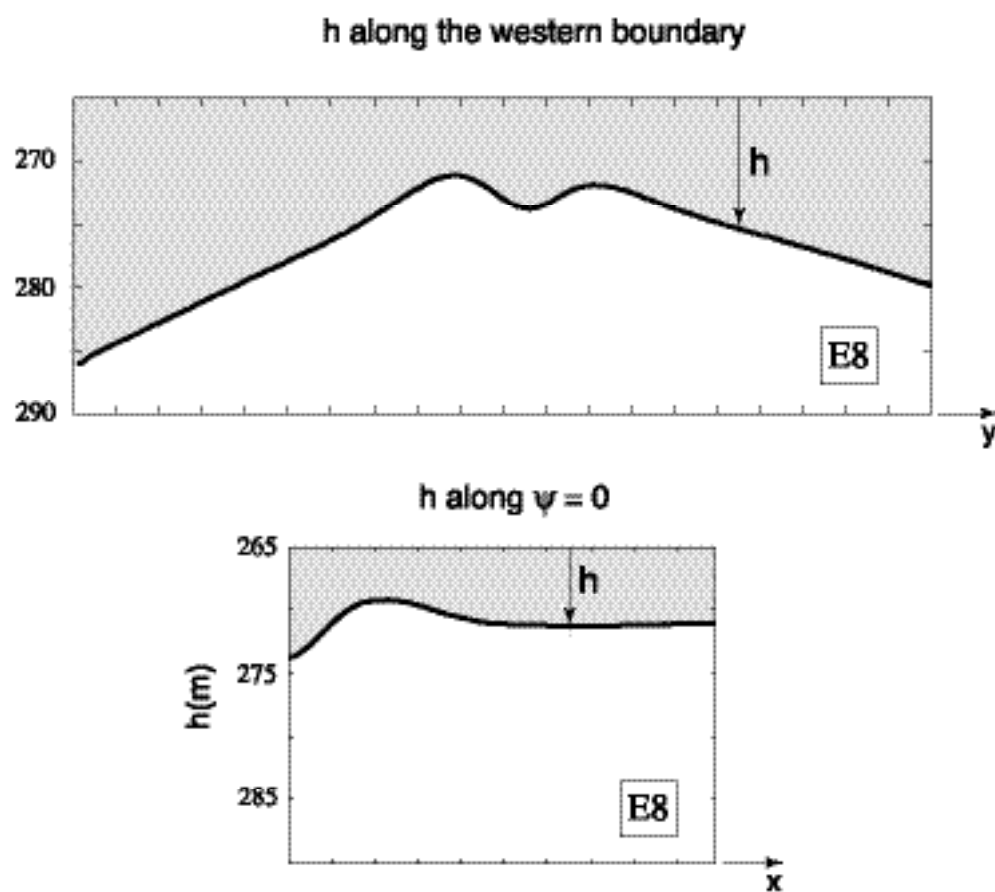


Fig. 14. Upper layer thickness h (in meters) along the western boundary (upper panel) and along the offshore branch of $\psi = 0$ (lower panel) for the collision experiment E8 at day 2500.

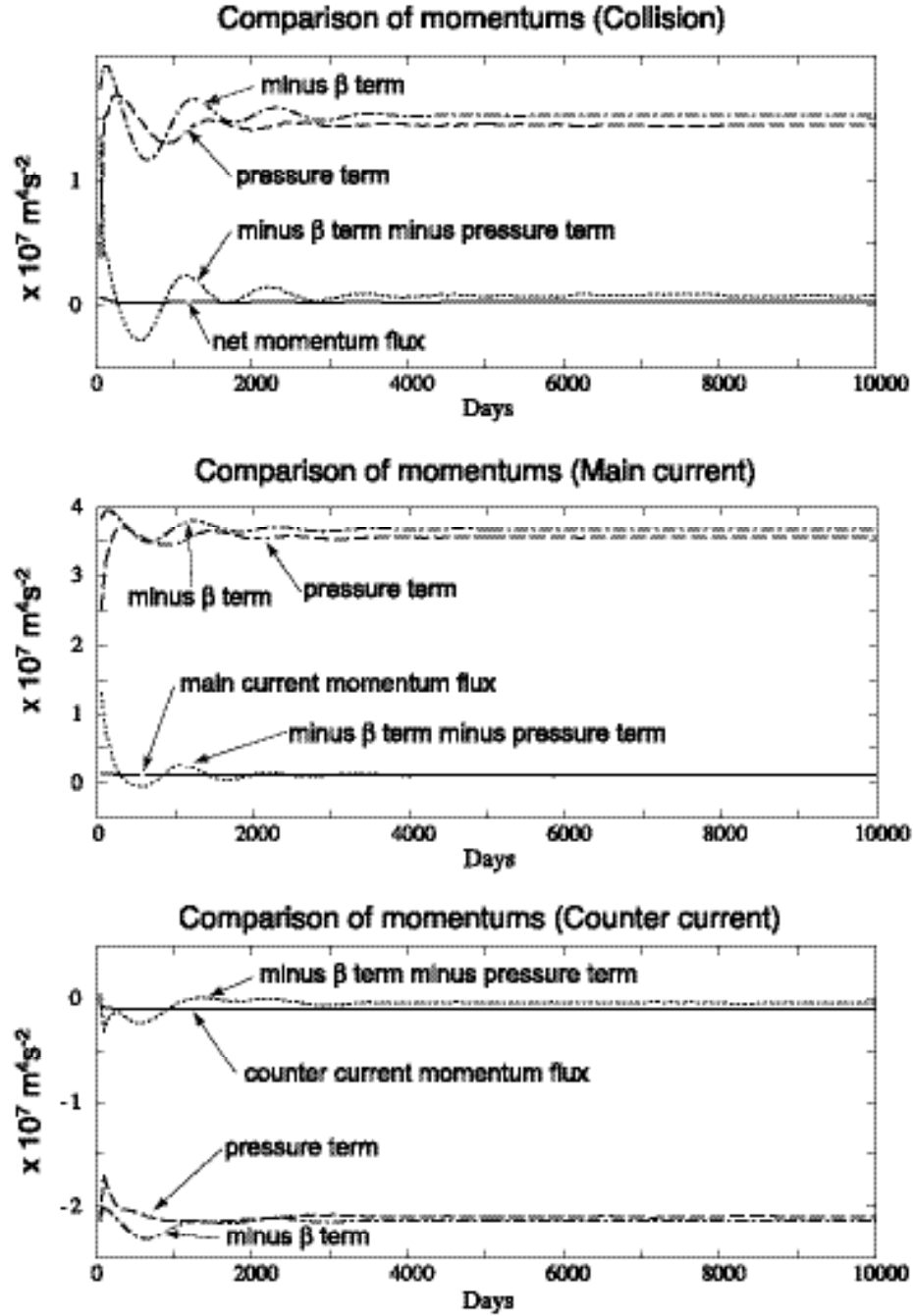


Fig. 15. Momentum balance for the area containing both colliding currents (upper panel); momentum balance for the area south of the zero streamfunction (middle panel); momentum balance for the area north of the zero streamfunction (lower panel). All terms were computed from experiment E8.

In the next section we shall apply the theory of the collision of opposing flowing WBC on a β -plane to the equatorial western Pacific; in this scenario, the NGCC is the “main current” and the MC is the “counter current.”

5. Discussion and summary

The sizes of the eddies as a function of their thickness are shown in **Fig. 16**. We see that in the linear limit ($H_0/H_n \ll 1$ for the cyclone and $H_0/H_s \ll 1$ for the anticyclone) both eddies disappear. This is consistent with Morey et al. (1999) who showed that both eddies do not exist in the linear case. **Fig. 16** also displays the ME and HE radii. We obtained these values by taking 26 Sv for the MC transport and 22 Sv for the NGCC transport. Approximately 11 Sv from the MC and 1 Sv from the SEC flow into the ITF. Using the values calculated by Nof (1996) for the offshore upper layer thicknesses of the colliding currents ($H_n = 97.83$ m for the MC and $H_s = 235.32$ m for the NGCC) we obtain $H_0 = 170$ m. Substituting the values of H_s , H_n , and H_0 into (11) and (14), we get a radius of $1.144 R_{dn}$ for the Mindanao eddy (where $R_{dn} = 110$ km and $\epsilon_n = 0.2$) and a radius of $1.52 R_{ds}$ for the Halmahera eddy (where $R_{ds} = 171$ km and $\epsilon_n = 0.3$). The estimated radii compare very well with the observations (Lukas et al. 1991; Kashino et al. 1999) of $1.136 R_{dn}$ (125 km) for the Mindanao eddy and between $1.37 R_{ds}$ and $1.46 R_{ds}$ (between 235 and 250 km) for the Halmahera eddy.

A fundamental question to ask is what causes the factor of two or so difference in the calculated eddy sizes. The answer is primarily nonlinearity which manifests itself via both large momentum fluxes and large difference between the thickness north and south of the NECC (H_n and H_s). Variations in the composition of the ITF would, of course, lead to changes in H_0 , and consequently, to variations in the relative sizes of the eddies.

Figs. 5 and 8 are the most elucidating of our analysis because they highlight the profound difference between an f -plane and a β -plane flow in a concave solid corner. In the f -plane case there is no eddy and the flow is diagonally symmetric while in the β -plane case there is lack of

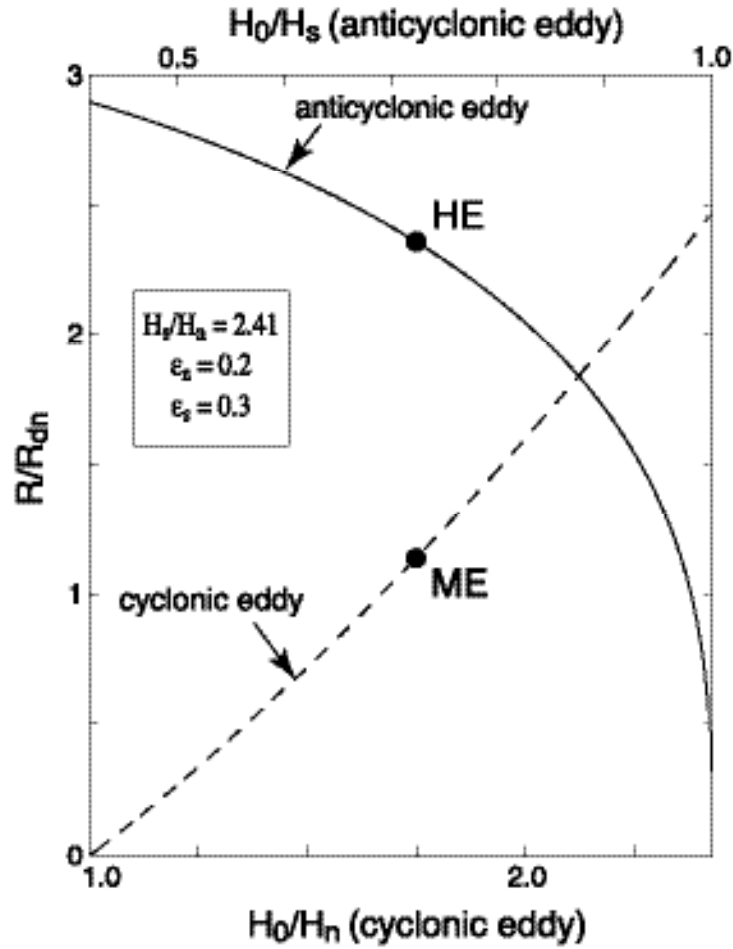


Fig. 16. The eddies' size as a function of the thickness ratio. The importance of nonlinearity is illustrated very vividly here as both the size of the cyclone and the anticyclones go to zero for small amplitude ($H_0/H_n \rightarrow 1$; $H_0/H_s \rightarrow 1$). Note that, on average, the HE is about twice as large as the ME (solid dots). The difference in size is due to the difference in nonlinearity. Note that, for $H_0/H_n > 2$, a larger HE implies a smaller ME and vice versa. Similarly, a weak NGCC (i.e., $H_0/H_s \rightarrow 1$) corresponds to no HE but large ME. Likewise, a weak MC (i.e., $H_0/H_n \rightarrow 1$) implies a small ME and large HE.

symmetry and a stationary eddy is attached to the curving flow. The momentum balance relations (6) for nonlinear WBC in concave solid corner shows that, on an f -plane, no eddy is necessary because the pressure force (produced by the difference between the upper layer thickness) balances the current's momentum flux. On a β -plane, on the other hand, the pressure force and the boundary current momentum force point in the same direction. Consequently, a permanent eddy is necessary to produce an opposing β -force leading to the required momentum balance. Nonlinearities are of equal fundamental importance because in the linear limit the boundary current momentum flux approaches zero so that no eddy is established. This is why Morey's et al. (1999) experiments show that there are no eddies when the nonlinearity is very small. In this sense our work has some similarity to the much larger (basin scale) re-circulation regions which also show up only when nonlinearity is present (i.e., they are not present in the limit of a frictional WBC and a Sverdrup interior). The lower bound estimate for the radius of the anticyclonic eddy is given by (13) and, similarly, the lower bound estimate for the radius of the cyclonic eddy is given by (14).

Applying the collision theory for the MC and NGCC in the western equatorial Pacific taking into account only the effective part of each current that participates in the collision process (i.e., excluding the parts that form the ITF), we estimate a diameter of 252 km for the ME and 520 km for the HE. These are in excellent agreement with the observed values of 250 km and between 470 and 500 km (Lukas et al. 1991; Kashino et al. 1999). We show that the difference between the two is primarily due to nonlinearity (**Fig. 16**), i.e., the stronger momentum flux of the NGCC relative to that of the MC. This nonlinearity manifests itself in a large difference in the thickness of the upper layer north and south of the NECC. An additional aspect that explains some (~50%) of the difference in the size is the difference in the Coriolis parameter (due to the different latitudes) which contributes to a larger Rossby radius in low latitudes.

Also, note that, according to our solution (**Fig. 15**), a weaker MC and stronger NGCC (i.e., $H_0/H_n \ll 1$) implies smaller ME and larger HE. Similarly, a stronger MC and a weaker

NGCC (i.e., $H_0/H_s \ll 1$) implies larger ME and smaller HE. This is in agreement with Waworantu (1999) statements that in Dec-Fed there is almost no HE but the ME is large (because the NGCC is very weak, i.e., $H_0/H_n \ll 1$). In the fall, on the other hand, the HE is strong and the ME is weak because the NGCC is strong (i.e., H_0/H_n is small). These results are also consistent with the reports of Lukas et al. 1991 and Kashino et al. 2001.

The above arguments show that the physical mechanism proposed here (i.e., that the eddies are necessary to balance the nonlinear momentum flux of their parent currents, the southward flowing MC and the northward flowing NGCC) are indeed responsible for the formation of the ME and HE. Of course, our model does not describe all of the oceanic details as it neglects motions below the upper layer as well as the inclination and complexity of the coastline. These could, no doubt, alter our results but our findings are nevertheless informative as they provide a first glance at the processes in question. We also point out that, although the ITF plays a secondary role in our theory, its existence is essential for the formation of the cross equatorial flow of the NGCC. Without the ITF there would be no collision and no eddies. Consequently, it is not surprising that variations in the ITF transport lead to variations of the relative sizes of the ME and the HE.

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Appendix. List of Symbols and Acronyms

B	Bernoulli function, $g'h + (u^2 + v^2)/2$
f	Coriolis parameter ($f_0 + \beta y$)
g'	Reduced gravity, $gD\rho/\rho$
h	Upper layer thickness
H	Undisturbed interior upper layer thickness (replaced by either H_n or H_s for the collision problem); see Fig. 3
H_0	Upper layer thickness on the zonal wall as $x \rightarrow \pm \infty$ • (Fig. 4)
$\hat{h}$	Upper layer thickness at the center of the anticyclonic eddy
H_{max}	Upper layer thickness at the corner (Fig. 4)
H_n, H_s	Upper layer thicknesses at fixed latitudes in the counter current side and main current side (Fig. 12)
L	Boundary current width (Fig. 2)
L_2	Width of square domain S (Figs. 2 and 3)
L_n	Counter current width (Fig. 12)
L_s	Main current width (Fig. 12)
R	Eddy radius
$\bar{R}$	Radius of the outcropped region in the cyclonic eddy (Fig. 11)
R_d	Rossby radius of deformation, $(g'H)^{1/2}/f_0$
R_{dn}	Rossby radius of deformation, $(g'H_n)^{1/2}/f_0$
R_{ds}	Rossby radius of deformation, $(g'H_s)^{1/2}/f_0$
R_{de}	Rossby deformation radius of the anticyclonic eddy, $(g'H_e)^{1/2}/f_0$
S	Integration area bounded by dashed rectangle in Figs. 2 and 3
S^+, S^-	Subsets of S such that $y \geq 0$ and $y < 0$, respectively
u, v	Velocity components in Cartesian coordinates
v_θ	Orbital velocity in the eddy

y_s, y_n	Latitudes of the southern and northern boundaries of S (Figs. 2, 3, and 12)
β	Variation of the Coriolis parameter with latitude
ε	Small parameter equal to $\beta R_d / f_0$
ϕ	Cartesian representation of the dividing streamline in the collision problem
$\rho, \Delta\rho$	Density and density difference between the layers
ν	Frictional coefficient
ψ	Streamfunction (defined by $\psi_y = -uh$; $\psi_x = -vh$)
ψ_∞	Limit of ψ as $x \rightarrow \infty$ •
$\hat{\psi}$	$(\psi - \psi_\infty)$
ADCP	Acoustic Doppler Current Profiler
HE	Halmahera eddy
ITF	Indonesian throughflow
MC	Mindanao Current
ME	Mindanao eddy
NECC	North Equatorial Counter Current
NGCC	New Guinea Coastal Current
NGCUC	New Guinea Coastal Undercurrent
SEC	South Equatorial Current
WBC	Western Boundary Current

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